Convolution Property →

If $x_1[n]$ & $x_2[n]$ are Two Descrete Time Signal

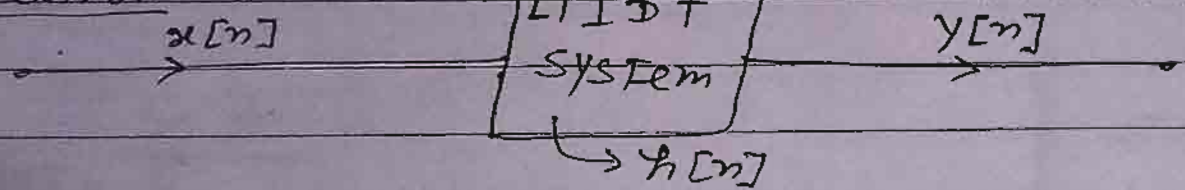
$$DTFT\{x_1[n]\} = X_1(e^{j\omega})$$

$$DTFT\{x_2[n]\} = X_2(e^{j\omega})$$

Then

$$\boxed{DTFT\{x_1[n] \otimes x_2[n]\} = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})}$$

APPLication →



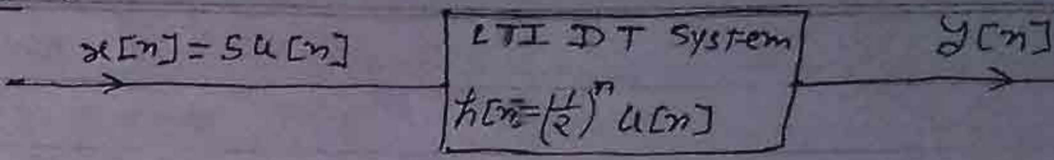
$$y[n] = x[n] \otimes h[n] \quad \text{——— (i)}$$

Taking DTFT Both side –

$$DTFT\{y[n]\} = DTFT\{x[n] \otimes h[n]\}$$

$$\boxed{Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})}$$

Question →



Determine $y[n]$ using DTFT.

Solⁿ → $\because x[n] = 5u[n]$

$$\therefore X(e^{j\omega}) = \text{DTFT } x[n] = \frac{5}{1 - e^{-j\omega}}$$

$$\because h[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$\therefore H(e^{j\omega}) = \text{DTFT}\{h[n]\} = \frac{1}{1 - \left(\frac{1}{2}\right) e^{-j\omega}}$$

we Know that, $Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$

$$\therefore Y(e^{j\omega}) = \frac{5}{(1 - e^{-j\omega})} \cdot \frac{1}{(1 - \frac{1}{2} e^{-j\omega})}$$

$$Y(e^{j\omega}) = \frac{A}{(1 - e^{-j\omega})} + \frac{B}{(1 - \frac{1}{2} e^{-j\omega})}$$

$$Y(e^{j\omega}) = \frac{10}{(1 - e^{-j\omega})} - \frac{5}{(1 - \frac{1}{2} e^{-j\omega})}$$

Taking IDTFT both Side -

$$y[n] = 10u[n] - 5\left(\frac{1}{2}\right)^n u[n]$$

→ Proof of Convolution Property →

We Know that —

(i) $DTFT\{x[n]\} = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

(ii) $x_1[n] \otimes x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$

(iii) $DTFT\{x[n-n_0]\} = \sum_{n=-\infty}^{\infty} x[n-n_0] e^{-j\omega n} = e^{-j\omega n_0} X(e^{j\omega})$

So we have $DTFT\{x_1[n] \otimes x_2[n]\} = \sum_{n=-\infty}^{\infty} \{x_1[n] \otimes x_2[n]\} e^{-j\omega n}$

$$= \sum_{n=-\infty}^{\infty} \left\{ \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k] \right\} e^{-j\omega n}$$

Now Interchanging the order of Summition —

$$DTFT\{x_1[n] \otimes x_2[n]\} = \sum_{k=-\infty}^{\infty} x_1[k] \left\{ \sum_{n=-\infty}^{\infty} x_2[n-k] e^{-j\omega n} \right\}$$

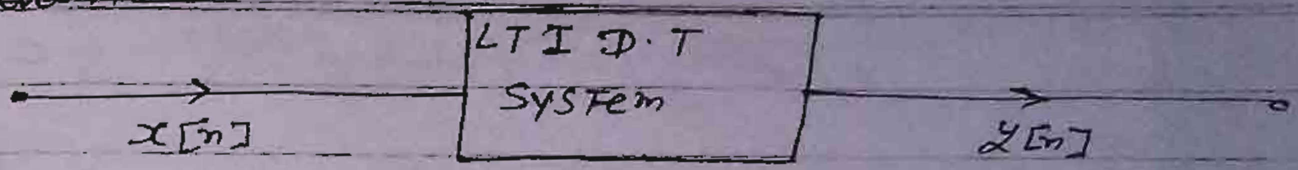
$$= \sum_{k=-\infty}^{\infty} x_1[k] \left\{ e^{-j\omega k} X_2(e^{j\omega}) \right\}$$

$$= X_2(e^{j\omega}) \sum_{k=-\infty}^{\infty} x_1[k] e^{-j\omega k}$$

$$= X_2(e^{j\omega})\, X_1(e^{j\omega})$$

$$\boxed{DTFT\{x_1[n] \otimes x_2[n]\} = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})}$$

Question →



The System is defined by –

$$y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = 3x[n]$$

Determine —

(i) Impulse Response of Given System (Transfer function)

(ii) Step Response of Given System.

(iii) Response of System when $x[n] = \left(\frac{1}{2}\right)^n u[n]$

Sol^n → Taken D.T.F.T both Side

$$\text{DTFT}\left\{y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2]\right\} = \text{DTFT}\{3x[n]\}$$

$$Y(e^{j\omega}) - \frac{5}{6} e^{-j\omega} Y(e^{j\omega}) + \frac{1}{6} e^{-j2\omega} Y(e^{j\omega}) = 3X(e^{j\omega})$$

$$Y(e^{j\omega}) \left\{1 - \frac{5}{6} e^{-j\omega} + \frac{1}{6} e^{-j2\omega}\right\} = 3X(e^{j\omega})$$

$$Y(e^{j\omega}) \left\{1 - \frac{3}{6} e^{-j\omega} - \frac{2}{6} e^{-j\omega} + \frac{1}{6} e^{-j2\omega}\right\} = 3X(e^{j\omega})$$

$$Y(e^{j\omega}) \left\{1\left(1 - \frac{1}{2} e^{-j\omega}\right) - \frac{1}{3} e^{-j\omega}\left(1 - \frac{1}{2} e^{-j\omega}\right)\right\} = 3X(e^{j\omega})$$

$$Y(e^{j\omega}) \left\{\left(1 - \frac{1}{2} e^{-j\omega}\right)\left(1 - \frac{1}{3} e^{-j\omega}\right)\right\} = 3X(e^{j\omega})$$

$$\Rightarrow \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{3}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})}$$

$$H(e^{j\omega}) = \frac{A}{(1-\frac{1}{2}e^{-j\omega})} + \frac{B}{(1-\frac{1}{3}e^{-j\omega})}$$

$$H(e^{j\omega}) = \frac{9}{(1-\frac{1}{2}e^{-j\omega})} - \frac{6}{(1-\frac{1}{3}e^{-j\omega})}$$

Now taken IDTFT Both side –

$$h[n] = 9\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{1}{3}\right)^n u[n]$$

→ For Impulse Response –

Put $x[n] = \delta[n]$

$\therefore X(e^{j\omega}) = 1$ ~~after taken DTFT both side~~

we know that, $y[n] = x[n] \otimes h[n]$

or $Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$

$$Y(e^{j\omega}) = 1 \cdot H(e^{j\omega})$$

taken IDTFT Both side – $y[n] = h[n]$

or $y[n] = 9\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{1}{3}\right)^n u[n]$

⇒ For Step Response -

Put $x[n] = u[n]$

$\therefore X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}$ after take DTFT both side

we know that -

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$

Putting the value -

· then $Y(e^{j\omega}) = \frac{1}{(1-e^{-j\omega})} \cdot \frac{3}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})}$

$$= \frac{3}{(1-e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})}$$

$$= \frac{A}{(1-e^{-j\omega})} + \frac{B}{(1-\frac{1}{2}e^{-j\omega})} + \frac{C}{(1-\frac{1}{3}e^{-j\omega})}$$

$$Y(e^{j\omega}) = \frac{9}{(1-e^{-j\omega})} - \frac{9}{(1-\frac{1}{2}e^{-j\omega})} + \frac{3}{(1-\frac{1}{3}e^{-j\omega})}$$

taken IDTFT Both side -

$$y[n] = 9u[n] - 9\left(\frac{1}{2}\right)^n u[n] + 3\left(\frac{1}{3}\right)^n u[n]$$

Response for $x[n] = \left(\frac{1}{2}\right)^n u[n]$

Let $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$X(e^{j\omega}) = \frac{1}{(1-\frac{1}{2}e^{-j\omega})}$$

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we Know that,

$$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw})$$

Putting the value -

then $$Y(e^{jw}) = \frac{1}{(1-\frac{1}{2}e^{-jw})} \cdot \frac{3}{(1-\frac{1}{2}e^{-jw})(1-\frac{1}{3}e^{-jw})}$$

$$Y(e^{jw}) = \frac{3}{(1-\frac{1}{3}e^{-jw})(1-\frac{1}{2}e^{-jw})^2}$$

By Partial fraction :-

$$Y(e^{jw}) = \frac{A}{(1-\frac{1}{3}e^{-jw})} + \frac{B}{(1-\frac{1}{2}e^{-jw})} + \frac{C}{(1-\frac{1}{2}e^{-jw})^2}$$

$$Y(e^{jw}) = \frac{12}{(1-\frac{1}{3}e^{-jw})} - \frac{18}{(1-\frac{1}{2}e^{-jw})} + \frac{9}{(1-\frac{1}{2}e^{-jw})^2}$$

$$Y(e^{jw}) = \frac{12}{(1-\frac{1}{3}e^{-jw})} - \frac{18}{(1-\frac{1}{2}e^{-jw})} + \frac{9(\frac{1}{2}e^{-jw})(2e^{-jw})}{(1-\frac{1}{2}e^{-jw})^2}$$

taken IDTFT both side -

$$y[n] = 12\left(\frac{1}{3}\right)^n u[n] - 18\left(\frac{1}{2}\right)^n u[n] + 18(n+1)\left(\frac{1}{2}\right)^{n+1} u[n+1]$$