Module - III (Statistical Techniques-I)

(1) Measures of central tendency :- Do your self from Youtube : Link Bhagwan Singh Vishwakarma.

Moments: Moments are statistical tools, used in statistical investigations. The moments of a distribution are the airthmetic means of the various powers of the deviations of items from some given number.

Moments About Mean (Central Moments)

(i) For individual Series :→ Let $x_1, x_2, x_3 ---- x_n$ be the data. Then, we know that,

$$\text{Mean } \bar{x} = \frac{\sum_{n=1}^{n} x_i}{n}$$

$\therefore$ r^{th} Moment μ_r about mean $\bar{x}$ is defined as

$$\mu_r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^r}{n} \; ; \quad r = 0, 1, 2 \,.....$$

Ex: Find first four moments for the following

x	3	6	8	10	18

Sol:- Here $n = 5$, $\Sigma x = 45$, $\bar{x} = 9$, $\bar{x} = \frac{\Sigma x_i}{n} = \frac{45}{5} = 9$

$\Sigma (x_i - \bar{x}) = 0$

$\Sigma (x_i - \bar{x})^2 = 128$

$\Sigma (x_i - \bar{x})^3 = 486$

$\Sigma (x_i - \bar{x})^4 = 7940$

$$\mu_r = \frac{\Sigma (x_i - \bar{x})^r}{n}$$

$$\therefore \mu_1 = \frac{\Sigma (x_i - \bar{x})^1}{n} = \frac{0}{5} = 0$$

$$\mu_2 = \frac{\Sigma (x_i - \bar{x})^2}{n} = \frac{128}{5} = 25.6$$

$$\mu_3 = \frac{\Sigma (x_i - \bar{x})^3}{n} = \frac{486}{5} = 97.2$$

$$\mu_4 = \frac{\Sigma (x_i - \bar{x})^3}{n} = \frac{7940}{5} = 1588$$

Moments about mean for a frequency distribution

$$\mu_r = \frac{1}{N} \sum_{i=1}^{N} f_i (x_i - \bar{x})^r, \quad \text{where } r = 0, 1, 2, \cdots$$

and $N = \sum_{i=1}^{n} f_i$

For ① $r = 0$, $\mu_0 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^0 = \frac{1}{N} \sum_{i=1}^{n} f_i$

$= \frac{1}{N} \times N = 1$

$\Rightarrow \boxed{\mu_0 = 1}$

For ② $r = 1$, $\mu_1 = \frac{1}{N} \sum f_i (x_i - \bar{x})$

$= \frac{1}{N} \cdot \sum f_i x_i - \bar{x} \cdot \frac{\sum f_i}{N} = \bar{x} - \bar{x} = 0$

$\therefore \boxed{\mu_1 = 0}$

For $r = 2$, $\mu_2 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^2 = (S.D.)^2$

$= \text{Variance}$

where S.D = Standerd Deviation

Similarly $\mu_3 = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^3$

$\mu_4 = \frac{1}{N} \sum_{i=1}^{N} f_i (x_i - \bar{x})^4$

and so on

Question: → Calculate $\mu_1, \mu_2, \mu_3, \mu_4$ for the following frequency distribution

Marks	5-15	15-25	25-35	35-45	45-55	55-65
No. of Students	10	20	25	20	15	15

Hint: → See Page No. 3.

$$\bar{x}\ (\text{Mean}) = \frac{\sum f_i x_i}{N} = \frac{3400}{100} = 34$$

$$\mu_r = \frac{\sum f_i (x_i - \bar{x})^r}{N},$$

$$r = 0, \quad \mu_0 = \frac{\sum f_i (x_i - \bar{x})^0}{N} = \frac{\sum f_i}{N} = 1$$

$$r = 1 \Rightarrow \mu_1 = \frac{\sum f_i (x_i - \bar{x})^1}{N} = \frac{0}{100} = 0$$

$$r = 2 \Rightarrow \mu_2 = \frac{\sum f_i (x_i - \bar{x})^2}{N} = \frac{21400}{100} = 214$$

$$r = 3 \Rightarrow \mu_3 = \frac{\sum f_i (x_i - \bar{x})^3}{N} = \frac{46800}{100} = 468$$

$$r = 4 \Rightarrow \mu_4 = \frac{\sum f_i (x_i - \bar{x})^4}{N} = \frac{9671200}{100} = 96712$$

Moments About an Arbitrary Number (Row Moments)

[(यादृच्छिक संख्या) = कोई भी संख्या]

If $x_1, x_2, x_3 \ldots x_n$ are the value of Variables x with corrosponding frequencies $f_1, f_2, f_3 \ldots f_n$ respectivly, Then the r^{th} moment μ_r' about the Number A defined as

$$\mu_r' = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - A)^r, \quad r = 0, 1, 2, 3 \ldots$$

$$\text{Where } N = \sum_{i=1}^{n} f_i$$

for

$$r = 0 \Rightarrow \mu_0' = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - A)^0 = 1$$

$$r = 1 \Rightarrow \mu_1' = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - A)^1$$

$$= \frac{1}{N} \sum f_i x_i - \frac{A}{N} \sum f_i = \bar{x} - A$$