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(ii) Harmonic Mean: H.M. is given by $\frac{1}{H} = \int_a^b \frac{1}{x} f(x) dx$ —— 1(a)

(iii) Geomatric Mean: G.M. is given by

$$\log G = \int_a^b \log x \cdot f(x)\, dx \quad \text{—— } 1(b)$$

(iv) Moment about origin $\nu_r = \int_a^b x^r \cdot f(x)\, dx$

(V) Moment about any point $x = A$

$$\mu'_r = \int_a^b (x-A)^r \cdot f(x)\, dx$$

(Vi) Moment about Mean $= \int_a^b (x - \text{Mean})^r \cdot f(x)\, dx$

From above we can find $\nu_1, \nu_2$ --- and $\mu_1, \mu_2$ ---

Variance, S.D., Mean, Median, & Mode.

(Vii) Median M is find by $\int_a^M f(x) dx = \int_M^b f(x) dx = \frac{1}{2}$

Question :→ A random variable $x$ has the density function

$$f(x) = k \cdot \frac{1}{1+x^2}, \quad -\infty < x < \infty$$

Determine k and the distribution function.

Sol. It will be a density function if.

$$\int_{-\infty}^{\infty} k \cdot \frac{1}{1+x^2} dx = 1 \Rightarrow k \cdot \pi = 1 \Rightarrow k = \frac{1}{\pi}$$

$$F(x) = \int \frac{1}{\pi} \cdot \frac{1}{1+x^2} dx = \frac{1}{\pi} \tan^{-1} x + c$$

But $F(-\infty)$ should be zero for distribution function.

$$\therefore \frac{1}{\pi}\left(-\frac{\pi}{2}\right) + c = 0 \Rightarrow c = \frac{1}{2}$$

$$\therefore F(x) = \frac{1}{\pi} \tan^{-1} x + \frac{1}{2} \text{ for } -\infty < x < \infty.$$

②

**Question:** → A random variable $x$ is distributed at random between the values 0 and 1 so that its probability density function is: $f(x) = kx^2(1-x^3)$, where $k$ is a constant. Find the value of $k$. Using this value of $k$, find its mean and variance.

**Solution:** → Since $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow k\int_0^1 (x^2 - x^5)\,dx = 1$

$$\Rightarrow k\left|\frac{x^3}{3} - \frac{x^6}{6}\right|_0^1 = 1 \Rightarrow \boxed{k = 6}$$

$$\text{Mean} = \mu_1' = \int_{-\infty}^{\infty} x f(x)\,dx = 6\int_0^1 (x^3 - x^6)\,dx = \frac{9}{14} = \text{Mean}$$

$$\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\,dx = 6\int_0^1 (x^4 - x^7)\,dx = \frac{9}{20}$$

$$\text{Variance} = \mu_2 = \mu_2' - \mu_1'^2 = \left[\frac{9}{20} - \left(\frac{9}{14}\right)^2\right] = \frac{9}{245} \text{ Ans.}$$

**Question:** A variable $x$ is distributed at random between the values 0 and 4 and its probability density function given by: $f(x) = kx^3(4-x)^2$.

**Solution:** →

$$\int_{-\infty}^{\infty} f(x)\,dx = 1, \quad k\int_0^4 x^3(4-x)^2\,dx = 1 \Rightarrow k = \frac{15}{1024}$$

$$\text{Mean } \mu_1' = \int_{-\infty}^{\infty} x f(x)\,dx = k\int_0^4 x^4(4-x^2)\,dx = \frac{16}{7}$$

$$\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\,dx = k\int_0^4 x^5(4-x^2)\,dx = \frac{40}{7}$$

$$\text{Variance} = \mu_2 = \mu_2' - \mu_1'^2 = \left\{\frac{40}{7} - \left(\frac{16}{7}\right)^2\right\} = \frac{24}{49}$$

∴ Standard deviation $= \sqrt{\text{Variance}}$

$$\text{S.D.} = \sqrt{\frac{24}{49}} = \frac{2\sqrt{6}}{7} \text{ Ans.}$$