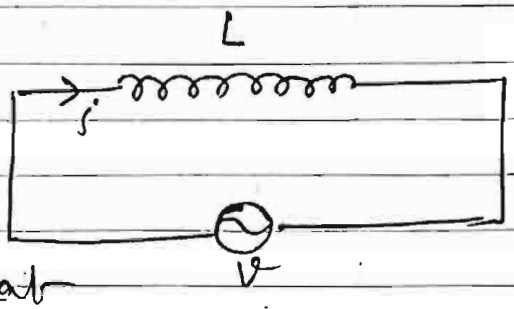
AC CIRCUIT containing pure Inductance :→



We know that

$$V = V_m \sin \omega t \quad \text{— (1)}$$

$$V = L \frac{di}{dt} \quad \text{(in case of inductance)}$$

$$V_m \sin \omega t = L \frac{di}{dt}$$

$$\frac{V_m}{L} \sin \omega t \, dt = di$$

integration of both sides.

$$\int \frac{V_m}{L} \sin \omega t \, dt = \int di$$

$$\frac{V_m}{L} \left[\frac{-\cos \omega t}{\omega} \right] = i$$

$$i = -\frac{V_m}{\omega L} \cos \omega t$$

$$i = -\frac{V_m}{\omega L} \sin \left(\frac{\pi}{2} - \omega t \right)$$

$V_m = I_m$

$$\frac{V_m}{\omega L} = I_m$$

$$i = -I_m \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{——— (2)}$$

Compare eqⁿ (1) and eqⁿ (2)

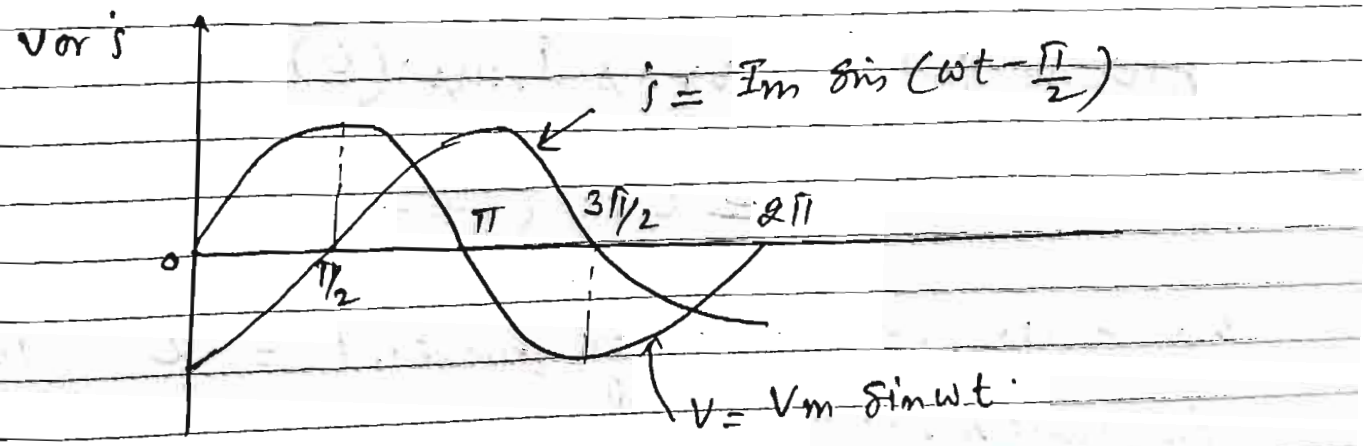
from eqⁿ (2) it is clear that phase angle of current is less $\frac{\pi}{2}$ as compared to voltage.

or. in case of inductance current is lagging behind 90° as compared to voltage.

Wave diagram

$v = V_m \sin\omega t$, $i = I_m \sin(\omega t - \pi/2)$

Put $\omega t = 0 \Rightarrow i = -I_m$.



In the case of pure inductance current lags behind applied voltage by 90°.

Notes

Appointment

Instantaneous power

$$p = vi$$

$$= V_m \sin\omega t \; I_m \sin(\omega t - \pi/2)$$

$$= -V_m \sin\omega t \; I_m \cos\omega t$$

$$= -V_m I_m \sin\omega t \cos\omega t$$

$$= -\frac{V_m I_m}{2} \; 2 \sin\omega t \cos\omega t$$

$$p = -\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t$$

$$\boxed{p = -V_{rms} I_{rms} \sin 2\omega t}$$

P_{av} (Average Power) $= -V_{rms} I_{rms} \cdot (0)$

$= 0$ (Zero)

The average value of $\sin 2\omega t = 0$ in sin wave form.

Inductive Reactance :→ in the above expression

$$I_m = \frac{V_m}{\omega L}$$

ωL → inductive Reactance

it is denoted by X_L i.e

$$X_L = \omega L$$

unit L → Henry, ω unit → radian/sec

X_L will be in ohms.

6. The voltage and current through a circuit element are.

$$v = 100 \sin(314t + 45^\circ) \text{ volts}$$
$$i = 10 \sin(314t + 315^\circ) \text{ amperes.}$$

(i) identify the circuit element.
(ii) find the value of inductor and inductive reactance
(iii) obtain expression for Power.

Ans from eq^n (1) and (2)

$$v = 100 \sin(314t + 45^\circ) \text{ volts.}$$
$$j = 10 \sin(314t + 315)$$
$$i = 10 \sin(314t + 315 - 360)$$
$$i = 10 \sin(314t - 45^\circ)$$

(i) so it is clear that current is lagging behind the applied voltage v by 90°
It means circuit element is an inductor.

(ii) $I_m = \frac{V_m}{\omega L}$ where $\omega L \rightarrow X_L$

$$X_L = \frac{V_m}{I_m} = \frac{100}{10} = 10\,\Omega.$$

$$L = \frac{X_L}{\omega} = \frac{10}{314} = 0.0318 \text{ H.}$$

(iii) $P = -V_{rms} I_{rms} \sin 2\omega t$

$$= -\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \sin 2\omega t$$

Notes Appointment

$$P = -\frac{100 \times 10}{2} \sin(2 \times 314t)$$

$$P = -500 \sin 628t$$ Ans