SIR CHHOTU RAM INSTITUTE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

ENGINEERING MECHANICS (BT-419)

NOTES ON LADDER FRICTION

Consider a ladder AB resting on the rough ground and leaning against a wall, as shown in Fig. 7.1.

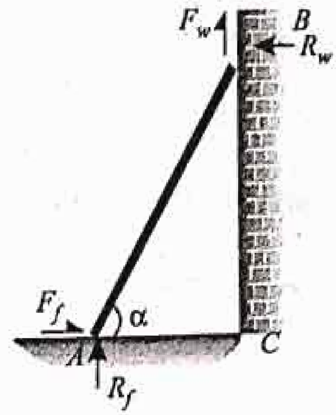

Fig. 7.1. Ladder friction

As the upper end of the ladder tends to slip downwards, therefore the direction of the force of friction between the ladder and the wall (F_w) will be upwards as shown in the figure. Similarly, as the lower end of the ladder tends to slip away from the wall, therefore the direction of the force of friction between the ladder and the floor (F_f) will be towards the wall as shown in the figure.

Since the system is in equilibrium, therefore the algebraic sum of the horizontal and vertical components of the forces must also be equal to zero.

Note: The normal reaction at the floor (R_f) will act perpendicular of the floor. Similarly, normal reaction of the wall (R_w) will also act perpendicular to the wall.

Example 7.1. *A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3.*

What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium in this position.

Solution. Given: Length of the ladder (l) = 3.25 m; Weight of the ladder (w) = 250 N; Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μ_f) = 0.3.

Frictional force acting on the ladder.

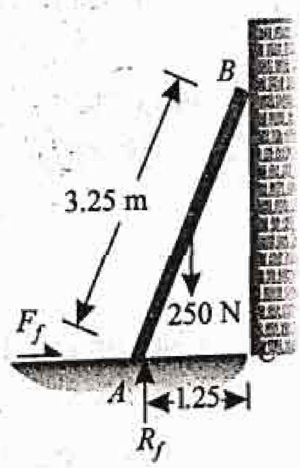

Fig. 7.2.

The forces acting on the ladder are shown in Fig. 7.2.

let F_f = Frictional force acting on the ladder at the Point of contact between the ladder and floor, and

R_f = Normal reaction at the floor.

Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall. Resolving the forces vertically,

$$R_f = 250 \text{ N}$$

From the geometry of the figure, we find that

$$BC = \sqrt{(3.25)^2 - (1.25)^2} = 3.0 \text{ m}$$

Taking moments about B and equating the same,

$$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625) = (250 \times 1.25) - 156.3 = 156.2 \text{ N}$$

$$\therefore \quad F_f = \frac{156.2}{3} = 52.1 \text{N} \quad \textbf{Ans.}$$

Equilibrium of the ladder

We know that the maximum force of friction available at the point of contact between the ladder and the floor

$$= \mu R_f = 0.3 \times 250 = 75 \text{ N}$$

Thus we see that the amount of the force of friction available at the point of contact (75 N) is more than the force of friction required for equilibrium (52.1 N). Therefore the ladder will remain in an equilibrium position. **Ans.**

Example 7.2. *A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder.*

Calculate the coefficient of friction between the ladder and the floor.

Solution. Given: Length of the ladder (l) = 5 m; Angle which the ladder makes with the horizontal (α) = 70°; Weight of the ladder (w_1) = 900 N; Weight of man (w_2) = 750 N and distance between the man and bottom of ladder = 1.5 m.

Forces acting on the ladder are shown in Fig. 7.3.

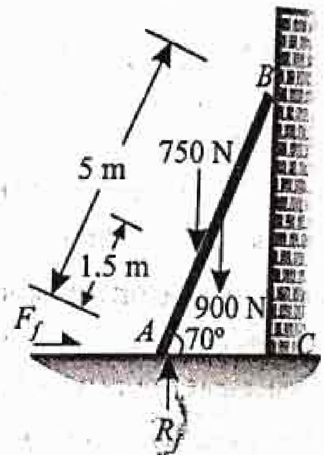

Fig. 7.3.

Let μ_f = Coefficient of friction between ladder and floor and

R_f = Normal reaction at the floor.

Resolving the forces vertically,

$$R_f = 900 + 750 = 1650 \text{ N} \quad ...(i)$$

∴ Force of friction at A

$$F_f = \mu_f \times R_f = \mu_f \times 1650 \quad ...(ii)$$

Now taking moments about B, and equating the same,

$$R_f \times 5 \sin 20° = (F_f \times 5 \cos 20°) + (900 \times 2.5 \sin 20°) + (750 \times 3.5 \sin 20°)$$

$$= (F_f \times 5 \cos 20°) + (4875 \sin 20°)$$

$$= (\mu_f \times 1650 \times 5 \cos 20°) + 4875 \sin 20°$$

and now substituting the values of R_f and F_f from equations (*i*) and (*ii*)

$$1650 \times 5 \sin 20° = (\mu_f \times 1650 \times 5 \cos 20°) + (4875 \sin 20°)$$

Dividing both sides by 5 sin 20°,

$$1650 = (\mu_f \times 1650 \cot 20°) + 975$$

$$= (\mu_f \times 1650 \times 2.7475) + 975 = 4533\,\mu_f + 975$$

$$\therefore \quad \mu_f = \frac{1650 - 975}{4533} = 0.15 \quad \textbf{Ans.}$$

Example 7.3. *A uniform ladder of 4 m length rests against a vertical wall with which it makes an angle of 45°. The coefficient of friction between the ladder and the wall is 0.4 and that between ladder and the floor is 0.5. If a man, whose weight is one-half of that of the ladder ascends it, how high will it be when the ladder slips?*

Solution. Given: Length of the ladder (l) = 4 μ; Angle which the ladder makes with the horizontal (α) = 45°; Coefficient of friction between the ladder and the wall (μ_w) = 0.4 and coefficient of friction between the ladder and the floor (μ_f) = 0.5.

The forces acting on the ladder are shown in Fig. 7.4.

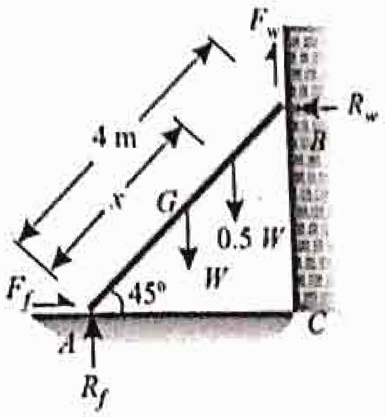

Fig. 7.4.

Let x = Distance between A and the man, when the ladder is at the point of slipping.

W = Weight of the ladder, and

R_f = Normal reaction at floor.

∴ Weight of the man

$$= \frac{W}{2} = 0.5\,W$$

We know that frictional force at the floor,

$$F_f = \mu_f\,R_f = 0.5\,R_f \qquad ...(i)$$

and frictional force at the wall,

$$F_w = \mu_w\,R_w = 0.4\,R_w \qquad ...(ii)$$

Resolving the forces vertically,

$$R_f + F_w = W + 0.5W = 1.5\,W \qquad ...(iii)$$

and now resolving the forces horizontally,

$$R_w = F_f = 0.5\,R_f \quad \text{or} \quad R_f = 2R_w$$

Now substituting the values of R_f and F_w in equation (*iii*),

$$2R_w + 0.4\,R_w = 1.5\,W$$

$$\therefore \quad R_w = \frac{1.5\,W}{2.4} = 0.625\,W$$

and

$$F_w = 0.4\,R_w = 0.4 \times 0.625\,W = 0.25\,W \qquad ...(iv)$$

Taking moments about A and equating the same,

$$(W \times 2\cos 45°) + (0.5\,W \times x\cos 45°)$$
$$= (R_w \times 4\sin 45°) + (F_w \times 4\cos 45°)$$

Substituting values of R_w *and* F_w from equations (*iii*) and (*iv*) in the above equation,

$$(W \times 2\cos 45°) + (0.5\,W \times x\cos 45°)$$
$$= (0.625\,W \times 4\sin 45°) + (0.25\,W \times 4\cos 45°)$$

Dividing both sides by ($W\sin 45°$),

$$2 + 0.5x = 2.5 + 1 = 3.5 \qquad ...(\because \sin 45° = \cos 45° = 0.707)$$

$$\therefore \quad x = \frac{3.5 - 2}{0.5} = 3.0\,\text{m} \qquad \textbf{Ans.}$$