## (b) Infinite Sequence Signal & Their ROC ⇒

### Infinite Sequence Signal :

(A) +ve Sided Infinite Sequence Signal

(B) −ve Sided "

(C) Both Sided "

$$S_\infty = a + ar + ar^2 + ar^3 + \dots ar^{n-1} + \dots + a(r)^\infty$$

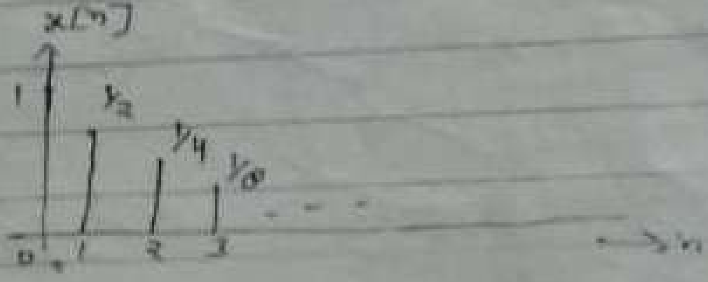
$$0 \text{ when } r<1 \qquad 1 \; r=1 \qquad \infty \; r>1$$

(i) $S_\infty = \dfrac{a}{1-r}$ It is Possible when $r<1$

(ii) $S_\infty = \infty$ when $r \geq 1$

### (A) +ve Sided Infinite Sequence Signal →

EXAMPLE⇒ $x[n] = \left(\frac{1}{2}\right)^n u[n] = \begin{cases} 0 & : n<0 \\ \left(\frac{1}{2}\right)^n & : n \geq 0 \end{cases}$




Determine $X(z)$ & Their ROC ?

Sol$^n$ ⇒ We know that. $Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n$$

$$X(z) = \dots + \left(\frac{1}{2} z^{-1}\right)^1 + \left(\frac{1}{2} z^{-1}\right)^2 + \dots \left(\frac{1}{2} z^{-1}\right)^\infty$$

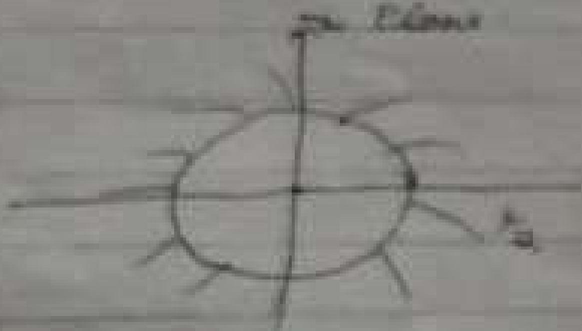
$$\boxed{X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})}}$$

but $\frac{1}{2}z^{-1} < 1$

or $\frac{1}{2}\frac{1}{z} < 1$

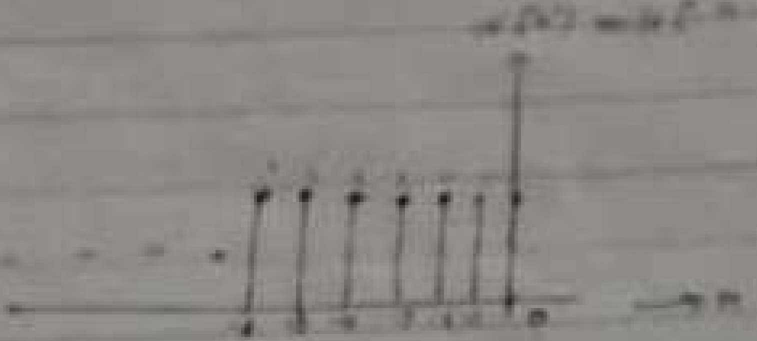
or $\boxed{z > \frac{1}{2}}$

ROC → ROC of +ve sided infinite sequence signal is out side the circle.



## (B) -Ve Sided Infinite Sequence Signal →

EXAMPLE → $x[n] = u[-n-1] = \begin{cases} 1 & : n<0 \\ 0 & : n \geq 0 \end{cases}$



Determine Z Transform & their ROC.

Sol$^n$ ⇒ We know that $Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

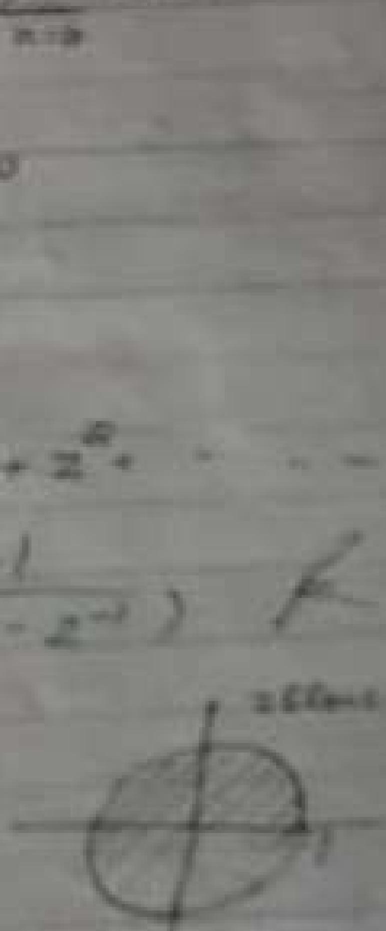
or $$X(z) = \sum_{n=-\infty}^{-1} x(n) z^{-n} + \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{-1} u[-n-1] z^{-n} + 0$$

$$= \sum_{n=-\infty}^{-1} 1 \cdot z^{-n}$$

$$= z + z^2 + z^3 + z^4 + z^5 + \dots$$

$$X(z) = \frac{z}{1-z} = \frac{-1}{(1-z^{-1})}$$



ROC ⇒ ROC of -ve sided infinite sequence signal is $|z|<1$

interior of inside the circle.

$$x[n] = -2^n u[-n-1]$$

<u>Question</u>

(i) Sketch the Signal.

(ii) Determine Z transform of given signal.

(iii) Determine ROC of given signal.

$$x[n] = -2^n u[-n-1] = \begin{cases} 0 & ; n>0 \\ -2^n & ; n \le 0 \end{cases}$$

Sol$^n$ ⇒ (i)


(ii) We Know That, $Z[x(n)] = X(Z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$$X(z) = \sum_{-\infty}^{-1} x[n] z^{-n} + \sum_{0}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{-1} -2^n u[-n-1] z^{-n} + 0$$

$$= -\sum_{n=-\infty}^{-1} 2^n z^{-n} \qquad \because u[-n-1] = 1$$

$$= -\left[\frac{1}{2}z + \frac{1}{4}z^2 + \frac{1}{8}z^3 + \cdots\right]$$

$$X(z) = -\left[\frac{\frac{1}{2}z}{1-\frac{1}{2}z}\right] = -\left[\frac{1}{\frac{1}{2^{-1}z} - 1}\right]$$

$$X(z) = \frac{1}{(1-2z^{-1})}$$

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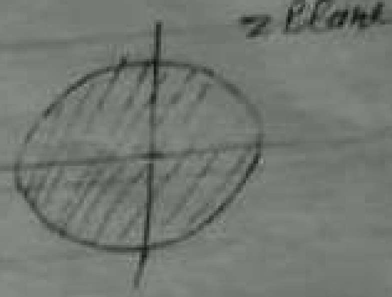
We Solve The Sum of This when Commen Ratio is Less than 1

ROC $\Rightarrow 2^{-1}z < 1$

or $\boxed{z < 2}$

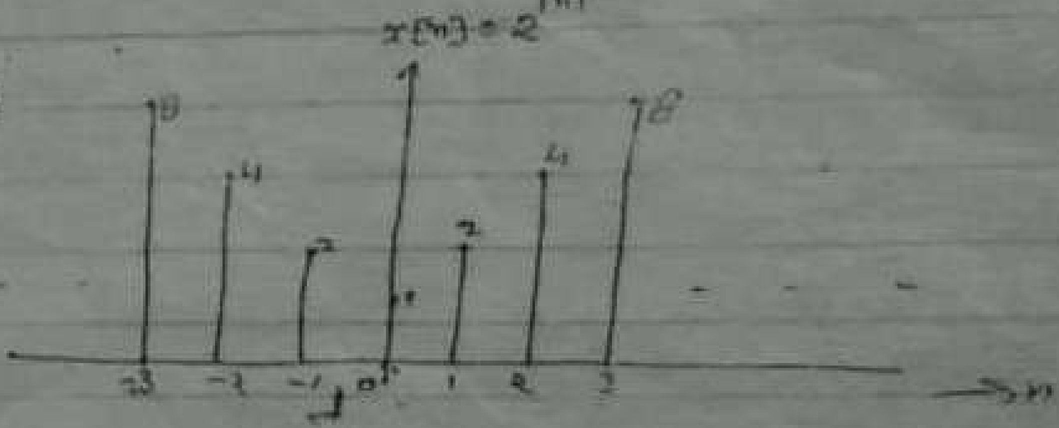



(C) <u>Both Sided Infinite Sequence Signal</u> $\Rightarrow$

<u>EXAMPLE</u> $\rightarrow$ $x[n] = 2^{|n|} = \begin{cases} 2^{-n} & ; n < 0 \\ 2^{n} & ; n \geq 0 \end{cases}$

(i) Sketch The Signal.
(ii) Determine Z transform of given Signal.
(iii) Determine ROC of given Signal.

<u>Sol^n</u> $\Rightarrow$ (i)




(ii) We Know That. $Z[x[n]] = X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$$X(z) = \sum_{n=-\infty}^{-1} x[n] z^{-n} + \sum_{n=0}^{\infty} x[n] z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{0} 2^{-n} z^{-n} + \sum_{n=0}^{\infty} 2^{n} z^{-n}$$

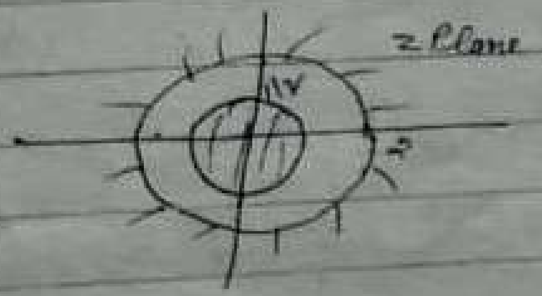
$$= (2z + 2^2 z^2 + 2^3 z^3 + \cdots) + (1 + 2 z^{-1} + 2^2 z^{-2} + \cdots)$$

$$X(z) = \left(\frac{2z}{1-2z}\right) + \left(\frac{1}{1-2z^{-1}}\right)$$

(iii) For ROC →

(i) $2z < 1$ or $z < \frac{1}{2}$

(ii) $2z^{-1} < 1$, $\frac{2}{z} < 1$, $z > 2$



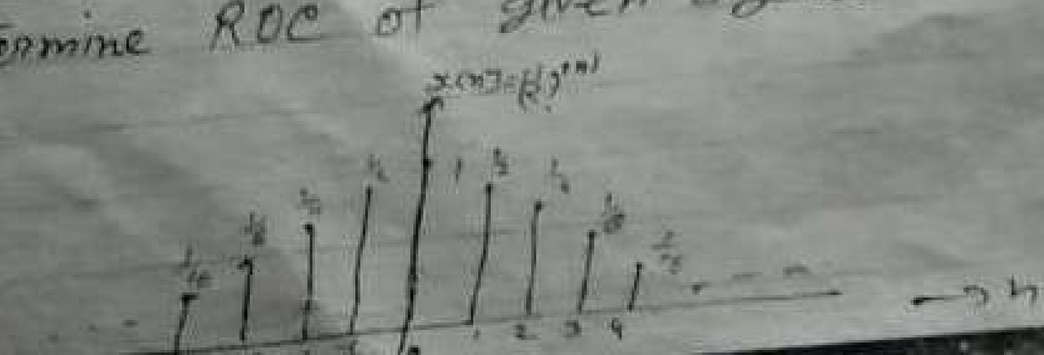

Question ⇒ $x[n] = \left(\frac{1}{2}\right)^{|n|} = \begin{cases} \left(\frac{1}{2}\right)^{-n}, & n \le 0 \\ \left(\frac{1}{2}\right)^{n}, & n \ge 0 \end{cases}$

(i) Sketch the Signal

(ii) Determine the Z Transform of given Signal

(iii) Determine ROC of given Signal.

Sol$^n$ →



(ii) We Know That. $Z[x[n]] = X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$

$$X(z) = \sum_{n=-\infty}^{-1} x[n]\, z^{-n} + \sum_{n=0}^{\infty} x[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{-1} \left(\tfrac{1}{2}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n} z^{-n}$$

$$= \left[\tfrac{1}{2}z + \left(\tfrac{1}{2}z\right)^2 + \left(\tfrac{1}{2}z\right)^3 + \cdots\right] + \left[1 + \tfrac{1}{2}z^{-1} + \tfrac{1}{2^2}z^{-2} + \cdots\right]$$

$$X(z) = \left(\frac{\frac{1}{2}z}{1-\frac{1}{2}z}\right) + \left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

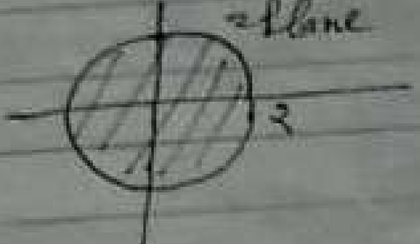
(iii) <u>For ROC →</u> (i) $\frac{1}{2}z < 1$

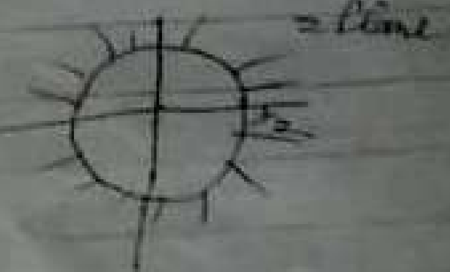
or $z < 2$

(ii) $\frac{1}{2}z^{-1} < 1$

$\frac{1}{2z} < 1$

$z > \frac{1}{2}$




then Complete ROC →