SIR CHHOTU RAM INSTITUTE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

## ENGINEERING MECHANICS (BT-419)

# NOTES ON MOMENT OF INERTIA

or second moment of mass. But all such second moments are broadly termed as moment of inertia. In this chapter, we shall discuss the moment of inertia of plane areas only.

## 15.2. Moment of Inertia of a Plane Area

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.

Let $a_1, a_2, a_3, ...$ = Areas of small elements, and

$r_1, r_2, r_3, ...$ = Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now the moment of inertia of the area,

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + ...$$
$$= \sum a r^2$$

## 15.3. Units of Moment of Inertia

As a matter of fact the units of moment of inertia of a plane area depend upon the units of the area and the length. *e.g.*,

1. If area is in $m^2$ and the length is also in m, the moment of inertia is expressed in $m^4$.
2. If area in $mm^2$ and the length is also in mm, then moment of inertia is expressed in $mm^4$.

## 15.4. Methods for Moment of Inertia

The moment of inertia of a plane area (or a body) may be found out by any one of the following two methods :

1. By Routh's rule  2. By Integration.

**Note :** The Routh's Rule is used for finding the moment of inertia of a plane area or a body of uniform thickness.

## 15.5. Moment of Inertia by Routh's Rule

The Routh's Rule states, if a body is symmetrical about three mutually perpendicular axes*, then the moment of inertia, about any one axis passing through its centre of gravity is given by:

$$I = \frac{A \text{ (or } M) \times S}{3} \quad \text{... (For a Square or Rectangular Lamina)}$$

$$I = \frac{A \text{ (or } M) \times S}{4} \quad \text{... (For a Circular or Elliptical Lamina)}$$

$$I = \frac{A \text{ (or } M) \times S}{5} \quad \text{... (For a Spherical Body)}$$

where $A$ = Area of the plane area

$M$ = Mass of the body, and

$S$ = Sum of the squares of the two semi-axis, other than the axis, about which the moment of inertia is required to be found out.

**Note :** This method has only academic importance and is rarely used in the field of science and technology these days. The reason for the same is that it is equally convenient to use the method of integration for the moment of inertia of a body.

* *i.e.*, *X-X* axis, *Y-Y* axis and *Z-Z* axis.

## 15.6. Moment of Inertia by Integration

The moment of inertia of an area may also be found out by the method of integration as discussed below:

Consider a plane figure, whose moment of inertia is required to be found out about $X$-$X$ axis and $Y$-$Y$ axis as shown in Fig 15.1. Let us divide the whole area into a no. of strips. Consider one of these strips.

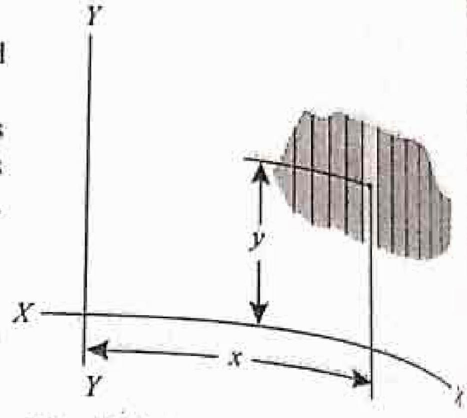

Fig. 15.1. Moment of Inertia by Integration.

Let $dA$ = Area of the strip

$x$ = Distance of the centre of gravity of the strip on $X$-$X$ axis and

$y$ = Distance of the centre of gravity of the strip on $Y$-$Y$ axis.

We know that the moment of inertia of the strip about $Y$-$Y$ axis

$$= dA \,.\, x^2$$

Now the moment of inertia of the whole area may be found out by integrating above equation. *i.e.*,

$$I_{YY} = \Sigma\, dA \,.\, x^2$$

Similarly $I_{XX} = \Sigma\, dA \,.\, y^2$

In the following pages, we shall discuss the applications of this method for finding out the moment of inertia of various cross-sections.

## 15.7. Moment of Inertia of a Rectangular Section

Consider a rectangular section $ABCD$ as shown in Fig. 15.2 whose moment of inertia is required to be found out.

Let $b$ = Width of the section and

$d$ = Depth of the section.

Now consider a strip $PQ$ of thickness $dy$ parallel to $X$-$X$ axis and at a distance $y$ from it as shown in the figure

$\therefore$ Area of the strip

$$= b.dy$$

We know that moment of inertia of the strip about $X$-$X$ axis,

$$= \text{Area} \times y^2 = (b.\ dy)\ y^2 = b.\ y^2.\ dy$$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina *i.e.* from $-\dfrac{d}{2}$ to $+\dfrac{d}{2}$,

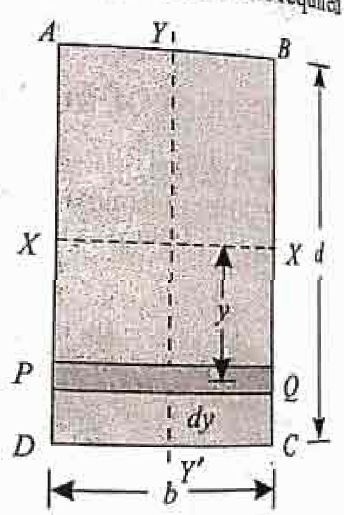

Fig. 15.2. Rectangular section.

* This may also be obtained by Routh's rule as discussed below :

$$I_{XX} = \frac{AS}{3} \qquad \text{...(for rectangular section)}$$

where area, $A = b \times d$ and sum of the square of semi axes $Y$-$Y$ and $Z$-$Z$,

$$S = \left(\frac{d}{2}\right)^2 + 0 = \frac{d^2}{4}$$

$$\therefore \quad I_{XX} = \frac{AS}{3} = \frac{(b \times d) \times \dfrac{d^2}{4}}{3} = \frac{bd^3}{12}$$

$$I_{XX} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b\int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \,.\, dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $I_{YY} = \dfrac{db^3}{12}$

**Note.** Cube is to be taken of the side, which is at right angles to the line of reference.

**Example 15.1.** *Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.*

**Solution.** Given: Width of the section ($b$) = 30 mm and depth of the section ($d$) = 40 mm.

We know that moment of inertia of the section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{30 \times (40)^3}{12} = 160 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly $$I_{YY} = \frac{db^3}{12} = \frac{40 \times (30)^3}{12} = 90 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

## 15.8. Moment of Inertia of a Hollow Rectangular Section

Consider a hollow rectangular section, in which *ABCD* is the main section and *EFGH* is the cut out section as shown in Fig 15.3

Let $b$ = Breadth of the outer rectangle,
$d$ = Depth of the outer rectangle and
$b_1$, $d_1$ = Corresponding values for the cut out rectangle.

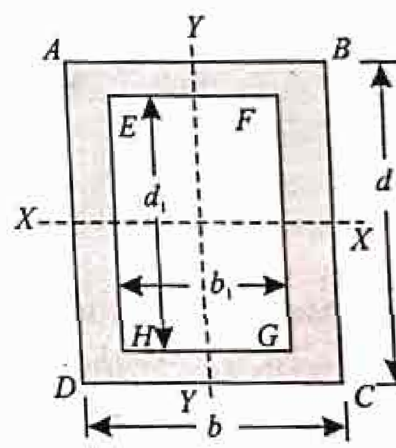


**Fig. 15.3.** Hollow rectangular section.

We know that the moment of inertia, of the outer rectangle *ABCD* about *X-X* axis

$$= \frac{bd^3}{12} \quad ...(i)$$

and moment of inertia of the cut out rectangle *EFGH* about *X-X* axis

$$= \frac{b_1 d_1^3}{12} \quad ...(ii)$$

∴ M.I. of the hollow rectangular section about *X-X* axis,

$$I_{XX} = \text{M.I. of rectangle } ABCD - \text{M.I. of rectangle } EFGH$$

$$= \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

Similarly, $$I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$$

**Note :** This relation holds good only if the centre of gravity of the main section as well as that of the cut out section coincide with each other.

**Example 15.2.** *Find the moment of inertia of a hollow rectangular section about its centre of gravity if the external dimensions are breadth 60 mm, depth 80 mm and internal dimensions are breadth 30 mm and depth 40 mm respectively.*

**Solution.** Given: External breadth $(b) = 60$ mm; External depth $(d) = 80$ mm ; Internal breadth $(b_1) = 30$ mm and internal depth $(d_1) = 40$ mm.

We know that moment of inertia of hollow rectangular section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12} = \frac{60\,(80)^3}{12} - \frac{30\,(40)^3}{12} = 2400 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly,
$$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12} = \frac{80\,(60)^3}{12} - \frac{40\,(30)^3}{12} = 1350 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

## 15.9. Theorem of Perpendicular Axis

It states, *If $I_{XX}$ and $I_{YY}$ be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia $I_{ZZ}$ about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:*

$$I_{ZZ} = I_{XX} + I_{YY}$$

**Proof :**

Consider a small lamina (*P*) of area *da* having co-ordinates as *x* and *y* along *OX* and *OY* two mutually perpendicular axes on a plane section as shown in Fig. 15.4.

Now consider a plane *OZ* perpendicular to *OX* and *OY*. Let (*r*) be the distance of the lamina (*P*) from Z-Z axis such that $OP = r$.

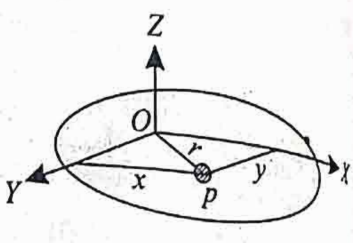

**Fig. 15.4.** Theorem of perpendicular axis.

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina *P* about *X-X* axis,

$$I_{XX} = da.\ y^2 \qquad \text{...}[\because I = \text{Area} \times (\text{Distance})^2]$$

Similarly, $$I_{YY} = da.\ x^2$$

and $$I_{ZZ} = da.\ r^2 = da\ (x^2 + y^2) \qquad \text{...}(\because r^2 = x^2 + y^2)$$
$$= da.\ x^2 + da.\ y^2 = I_{YY} + I_{XX}$$

## 15.10. Moment of Inertia of a Circular Section

Consider a circle *ABCD* of radius (*r*) with centre *O* and X-X' and Y-Y' be two axes of reference through *O* as shown in Fig. 15.5.

Now consider an elementary ring of radius *x* and thickness *dx*. Therefore area of the ring,

$$da = 2\,\pi\,x.\ dx$$

and moment of inertia of ring, about *X-X* axis or *Y-Y* axis

$$= \text{Area} \times (\text{Distance})^2$$
$$= 2\,\pi\,x.\ dx \times x^2$$
$$= 2\,\pi\,x^3.\ dx$$

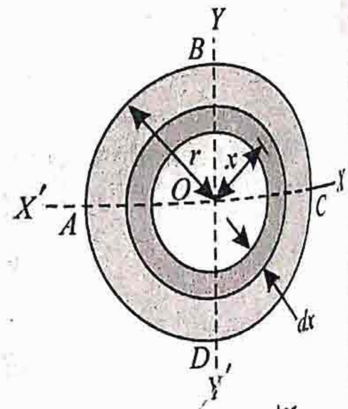

**Fig. 15.5.** Circular

Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.* from 0 to *r*.

$$\therefore \quad I_{ZZ} = \int_0^r 2\pi x^3\ .\ dx = 2\pi \int_0^r x^3\ .\ dx$$

$$I_{ZZ} = 2\pi \left[\frac{x^4}{4}\right]_0^r = \frac{\pi}{2}(r)^4 = \frac{\pi}{32}(d)^4 \qquad \ldots\left(\text{substituting } r = \frac{d}{2}\right)$$

We know from the Theorem of Perpendicular Axis that

$$I_{XX} + I_{YY} = I_{ZZ}$$

$$*\, I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{2} \times \frac{\pi}{32}(d)^4 = \frac{\pi}{64}(d)^4$$

**(UPTU 2009)**

**Example 15.3.** *Find the moment of inertia of a circular section of 50 mm diameter about an axis passing through its centre.*

**Solution.** Given: Diameter $(d) = 50$ mm

We know that moment of inertia of the circular section about an axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(d)^4 = \frac{\pi}{64} \times (50)^4 = 307 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

## 15.11. Moment of Inertia of a Hollow Circular Section

Consider a hollow circular section as shown in Fig.15.6, whose moment of inertia is required to be found out.

Let $D$ = Diameter of the main circle, and

$d$ = Diameter of the cut out circle.

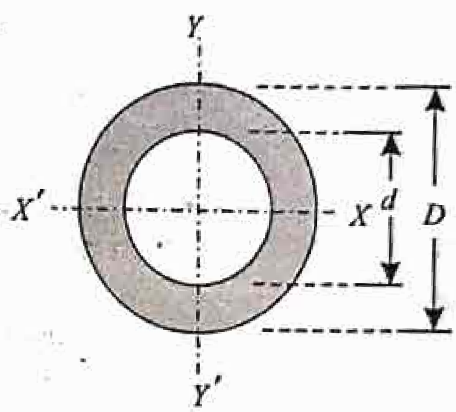

Fig. 15.6. Hollow circular section.

We know that the moment of inertia of the main circle about $X$-$X$ axis

$$= \frac{\pi}{64}(D)^4$$

and moment of inertia of the cut-out circle about $X$-$X$ axis

$$= \frac{\pi}{64}(d)^4$$

∴ Moment of inertia of the hollow circular section about $X$-$X$ axis,

$I_{XX}$ = Moment of inertia of main circle – Moment of inertia of cut out circle,

$$= \frac{\pi}{64}(D)^4 - \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(D^4 - d^4)$$

Similarly, $$I_{YY} = \frac{\pi}{64}(D^4 - d^4)$$

**Note :** This relation holds good only if the centre of the main circular section as well as that of the cut out circular section coincide with each other.

* This may also be obtained by Routh's rule as discussed below

$$I_{XX} = \frac{AS}{4} \qquad \text{(for circular section)}$$

Where area, $A = \frac{\pi}{4} \times d^2$ and sum of the square of semi axis $Y$-$Y$ and $Z$-$Z$,

$$S = \left(\frac{d}{2}\right)^2 + 0 = \frac{d^2}{4}$$

$$I_{XX} = \frac{AS}{4} = \frac{\left[\frac{\pi}{4} \times d^2\right] \times \frac{d^2}{4}}{4} = \frac{\pi}{64}(d)^4$$

**Example 15.4.** *A hollow circular section has an external diameter of 80 mm and internal diameter of 60 mm. Find its moment of inertia about the horizontal axis passing through its centre.*

**Solution.** Given : External diameter $(D) = 80$ mm and internal diameter $(d) = 60$ mm.

We know that moment of inertia of the hollow circular section about the horizontal axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}[(80)^4 - (60)^4] = 1374 \times 10^3 \text{ mm}^4$$

**Ans.**

## 15.12. Theorem of Parallel Axis

It states, *If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by $I_G$, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:*

$$I_{AB} = I_G + ah^2$$

where $I_{AB}$ = Moment of inertia of the area about an axis $AB$,

$I_G$ = Moment of Inertia of the area about its centre of gravity

$a$ = Area of the section, and

$h$ = Distance between centre of gravity of the section and axis $AB$.

### Proof

Consider a strip of a circle, whose moment of inertia is required to be found out about a line $AB$ as shown in Fig. 15.7.

Let $\delta a$ = Area of the strip

$y$ = Distance of the strip from the centre of gravity the section and

$h$ = Distance between centre of gravity of the section and the axis $AB$.

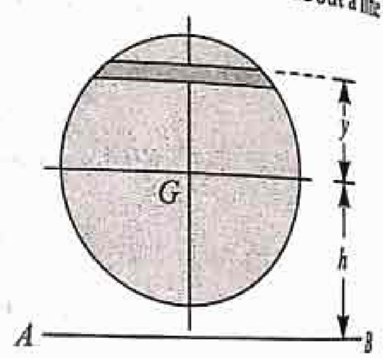

**Fig. 15.7.** Theorem of parallel axis.

We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section

$$= \delta a . y^2$$

and moment of inertia of the whole section about an axis passing through its centre of gravity,

$$I_G = \Sigma\, \delta a . y^2$$

∴ Moment of inertia of the section about the axis $AB$,

$$I_{AB} = \Sigma\, \delta a\,(h + y)^2 = \Sigma\, \delta a\,(h^2 + y^2 + 2\,h\,y)$$

$$= (\Sigma\, h^2 . \delta a) + (\Sigma\, y^2 . \delta a) + (\Sigma\, 2\,h\,y\,.\,\delta a)$$

$$= a\,h^2 + I_G + 0$$

It may be noted that $\Sigma\, h^2 . \delta a = a\,h^2$ and $\Sigma\, y^2 . \delta a = I_G$ [as per equation (*i*) above] and $\Sigma\, \delta a.y$ is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to $a.\bar{y}$, where $\bar{y}$ is the distance between the section and the axis passing through the centre of gravity, which obviously is zero.

## 15.13. Moment of Inertia of a Triangular Section

Consider a triangular section $ABC$ whose moment of inertia is required to be found out.

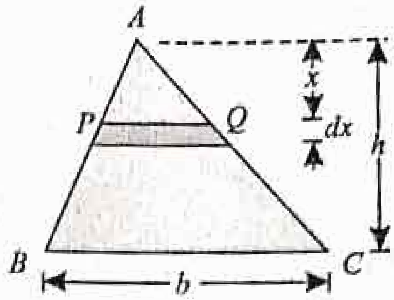

Fig. 15.8. Triangular section.

Let $b$ = Base of the triangular section and

$h$ = Height of the triangular section.

Now consider a small strip $PQ$ of thickness $dx$ at a distance of $x$ from the vertex $A$ as shown in Fig. 15.8. From the geometry of the figure, we find that the two triangles $APQ$ and $ABC$ are similar. Therefore

$$\frac{PQ}{BC} = \frac{x}{h} \quad \text{or} \quad PQ = \frac{BC.x}{h} = \frac{bx}{h} \qquad (\because BC = \text{base} = b)$$

We know that area of the strip $PQ$

$$= \frac{bx}{h}.dx$$

and moment of inertia of the strip about the base $BC$

$$= \text{Area} \times (\text{Distance})^2 = \frac{bx}{h} dx\,(h-x)^2 = \frac{bx}{h}(h-x)^2\,dx$$

Now moment of inertia of the whole triangular section may be found out by integrating the above equation for the whole height of the triangle *i.e.*, from 0 to $h$.

$$I_{BC} = \int_0^h \frac{bx}{h}(h-x)^2\,dx$$

$$= \frac{b}{h}\int_0^h x\,(h^2 + x^2 - 2hx)\,dx$$

$$= \frac{b}{h}\int_0^h (xh^2 + x^3 - 2hx^2)\,dx$$

$$= \frac{b}{h}\left[\frac{x^2h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3}\right]_0^h = \frac{bh^3}{12}$$

We know that distance between centre of gravity of the triangular section and base $BC$,

$$d = \frac{h}{3}$$

∴ Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to $X$-$X$ axis,

$$...(\because I_{XX} = I_G + a\,h^2)$$

$$I_G = I_{BC} - ad^2$$

$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

**Notes : 1.** The moment of inertia of section about an axis through its vertex and parallel to the base

$$= I_G + ad^2 = \frac{bh^3}{36} + \left(\frac{bh}{2}\right)\left(\frac{2h}{3}\right)^2 = \frac{9bh^3}{36} = \frac{bh^3}{4}$$

**2.** This relation holds good for any type of triangle.

**Example. 15.5.** *An isosceles triangular section ABC has base width 80 mm and height 60 mm. Determine the moment of inertia of the section about the centre of gravity of the section and the base BC.*

**Solution.** Given : Base width ($b$) = 80 mm and height ($h$) = 60 mm.

*Moment of inertia about the centre of gravity of the section*

We know that moment of inertia of triangular section about its centre of gravity,

$$I_G = \frac{bh^3}{36} = \frac{80 \times (60)^3}{36} = 480 \times 10^3 \text{ mm}^4$$

*Moment of inertia about the base BC*

We also know that moment of inertia of triangular section about the base $BC$,

$$I_{BC} = \frac{bh^3}{12} = \frac{80 \times (60)^3}{12} = 1440 \times 10^3 \text{ mm}^4$$

**Exmple 15.6.** *Hollow triangular section shown in Fig. 15.9 is symmetrical about its vertical axis.*

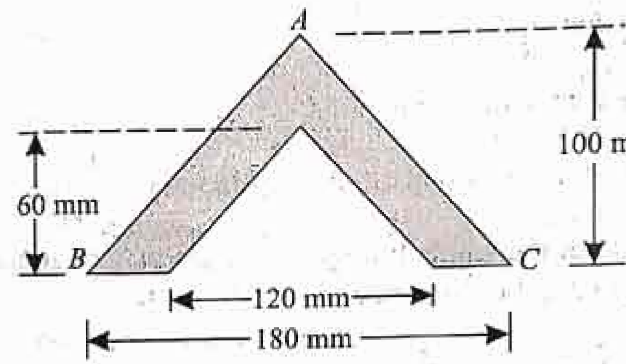


**Fig. 15.9.**

*Find the moment of inertia of the section about the base BC.*

**Solution.** Given : Base width of main triangle ($B$) = 180 mm; Base width of cut out triangle ($b$) = 120 mm; Height of main triangle ($H$) = 100 mm and height of cut out triangle ($h$) = 60 mm.

We know that moment of inertia of the triangular, section about the base $BC$,

$$I_{BC} = \frac{BH^3}{12} - \frac{bh^3}{12} = \frac{180 \times (100)^3}{12} - \frac{120 \times (60)^3}{12} \text{ mm}^4$$

$$= (15 \times 10^6) - (2.16 \times 10^6) = 12.84 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

## 15.14. Moment of Inertia of a Semicircular Section

Consider a semicircular section $ABC$ whose moment of inertia is required to be found out as shown in Fig. 15.10.

Let $r$ = Radius of the semicircle.

We know that moment of inertia of the semicircular section about the base $AC$ is equal to half the moment of inertia of the circular section about $AC$. Therefore moment of inertia of the semicircular section $ABC$ about the base $AC$,

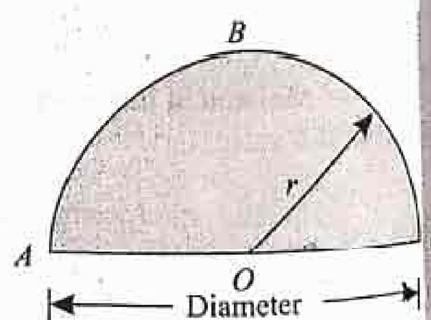


**Fig. 15.10.** Semicircular section ABC.

$$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393\, r^4$$

We also know that area of semicircular section,

$$a = \frac{1}{2} \times \pi r^2 \;\; \frac{\pi r^2}{2}$$

and distance between centre of gravity of the section and the base $AC$,

$$h = \frac{4r}{3\pi}$$

$\therefore$ Moment of inertia of the section through its centre of gravity and parallel to $x$-$x$ axis,

$$I_G = I_{AC} - ah^2 = \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2\right]$$

$$= \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{8}{9\pi} \times (r)^4\right] = 0.11\, r^4$$

**Note.** The moment of inertia about $y$-$y$ axis will be the same as that about the base $AC$ *i.e.*, $0.393\, r^4$.

**Example 15.7.** *Determine the moment of inertia of a semicircular section of 100 mm diameter about its centre of gravity and parallel to X-X and Y-Y axes.*

**Solution.** Given: Diameter of the section ($d$) = 100 mm or radius ($r$) = 50 mm

*Moment of inertia of the section about its centre of gravity and parallel to X-X axis*

We know that moment of inertia of the semicircular section about its centre of gravity and parallel to $X$-$X$ axis,

$$I_{XX} = 0.11\, r^4 = 0.11 \times (50)^4 = 687.5 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

*Moment of inertia of the section about its centre of gravity and parallel to Y-Y axis.*

We also know that moment of inertia of the semicircular section about its centre of gravity and parallel to $Y$-$Y$ axis.

$$I_{YY} = 0.393\, r^4 = 0.393 \times (50)^4 = 2456 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

**Example 15.8.** *A hollow semicircular section has its outer and inner diameter of 200 mm and 120 mm respectively as shown in Fig. 15.11.*

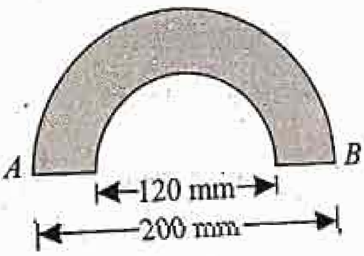


**Fig. 15.11.**

*What is its moment of inertia about the base AB ?*

**Solution.** Given: Outer diameter ($D$) = 200 mm or Outer Radius ($R$) = 100 mm and inner diameter ($d$) = 120 mm or inner radius ($r$) = 60 mm.

We know that moment of inertia of the hollow semicircular section about the base $AB$,

$$I_{AB} = 0.393\,(R^4 - r^4) = 0.393\,[(100)^4 - (60)^4] = 34.21 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

## EXERCISE 15.1

1. Find the moment of inertia of a rectangular section 60 mm wide and 40 mm deep about its centre of gravity. [**Ans.** $I_{XX} = 320 \times 10^3$ mm$^4$ ; $I_{YY} = 720 \times 10^3$ mm$^4$]
2. Find the moment of inertia of a hollow rectangular section about its centre of gravity, if the external dimensions are 40 mm deep and 30 mm wide and internal dimensions are 25 mm deep and 15 mm wide. [**Ans.** $I_{XX} = 140\,470$ mm$^4$ ; $I_{YY} = 82\,970$ mm$^4$]

3. Find the moment of inertia of a circular section of 20 mm diameter through its centre of gravity. [Ans. 7854 mm$^4$]
4. Calculate the moment of inertia of a hollow circular section of external and internal diameters 100 mm and 80 mm respectively about an axis passing through its centroid. [Ans. $2.898 \times 10^6$ mm$^4$]
5. Find the moment of inertia of a triangular section having 50 mm base and 60 mm height about an axis through its centre of gravity and base. [Ans. $300 \times 10^3$ mm$^4$; $900 \times 10^3$ mm$^4$]
6. Find the moment of inertia of a semicircular section of 30 mm radius about its centre of gravity and parallel to X-X and Y-Y axes. [Ans. 89 100 mm$^4$ ; 381 330 mm$^4$]

## 15.15. Moment of Inertia of a Composite Section

The moment of inertia of a composite section may be found out by the following steps :

1. First of all, split up the given section into plane areas (*i.e.*, rectangular, triangular, circular etc., and find the centre of gravity of the section).
2. Find the moments of inertia of these areas about their respective centres of gravity.
3. Now transfer these moment of inertia about the required axis (*AB*) by the Theorem of Parallel Axis, *i.e.*,

$$I_{AB} = I_G + ah^2$$

where $I_G$ = Moment of inertia of a section about its centre of gravity and parallel to the axis.
$a$ = Area of the section,
$h$ = Distance between the required axis and centre of gravity of the section.

4. The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

**Example 15.9.** *Figure 15.12 shows an area ABCDEF.*

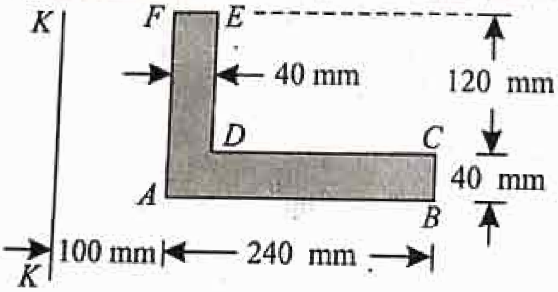


**Fig. 15.12.**

*Compute the moment of inertia of the above area about axis K-K.*

**Solution.** As the moment of inertia is required to be found out about the axis *K-K*, therefore there is no need of finding out the centre of gravity of the area.

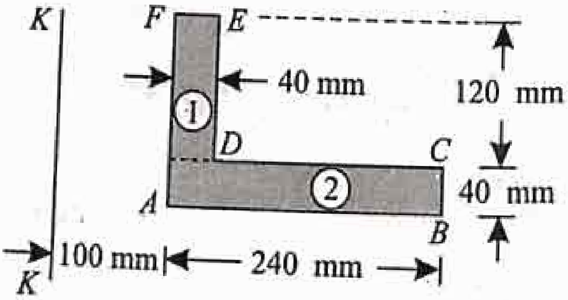


**Fig. 15.13.**

Let us split up the area into two rectangles 1 and 2 as shown in Fig. 15.13.

We know that moment of inertia of section (1) about its centre of gravity and parallel to axis *K-K*,

$$I_{G1} = \frac{120 \times (40)^3}{12} = 640 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of section (1) and axis *K-K*,

$$h_1 = 100 + \frac{40}{2} = 120 \text{ mm}$$

∴ Moment of inertia of section (1) about axis *K-K*

$$= I_{G1} + a_1 h_1^2 = (640 \times 10^3) + [(120 \times 40) \times (120)^2] = 69.76 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of section (2) about its centre of gravity and parallel to axis *K-K*,

$$I_{G2} = \frac{40 \times (240)^3}{12} = 46.08 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of section (2) and axis *K-K*,

$$h_2 = 100 + \frac{240}{2} = 220 \text{ mm}$$

∴ Moment of inertia of section (2) about the axis *K-K*,

$$= I_{G2} + a_2 h_2^2 = (46.08 \times 10^6) + [(240 \times 40) \times (220)^2] = 510.72 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole area about axis *K-K*,

$$I_{KK} = (69.76 \times 10^6) + (510.72 \times 10^6) = 580.48 \times 10^6 \text{ mm}^4$$ **Ans.**

**Example 15.10.** *Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.*

**Solution.** The given *T*-section is shown in Fig. 15.14.

First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles *viz.*, 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

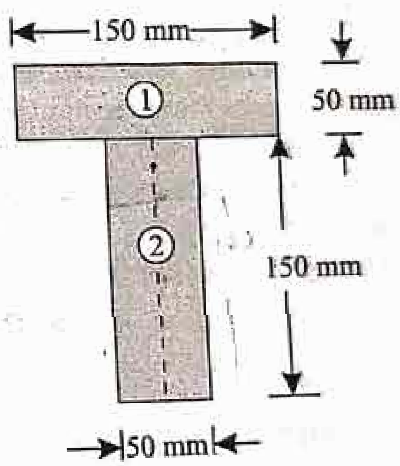

Fig. 15.14.

(*i*) *Rectangle* (1)

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

(*ii*) *Rectangle* (2)

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_2 = \frac{150}{2} = 75 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

*Moment of inertia about X-X axis*

We also know that M.I. of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis.

$$I_{G1} = \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and *X-X* axis,

$$h_1 = 175 - 125 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis

$$I_{G1} + a_1 h_1^2 = (1.5625 \times 10^6) + [7500 \times (50)^2] = 20.3125 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis.

$$I_{G2} = \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = 125 - 75 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis

$$= I_{G2} + a_2 h_2^2 = (14.0625 \times 10^6) + [7500 \times (50)^2] = 32.8125 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (20.3125 \times 10^6) + (32.8125 \times 10^6) = 53.125 \times 10^6 \text{ mm}^4$$ **Ans.**

*Moment of inertia about Y-Y axis*

We know that M.I. of rectangle (1) about Y-Y axis

$$= \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and moment of inertia of rectangle (2) about Y-Y axis,

$$= \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about Y-Y axis,

$$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 \text{ mm}^4$$ **Ans.**