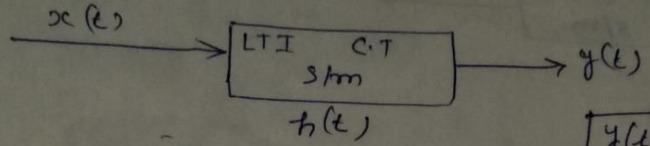
UNIT -2

LINEAR SHIFT INVARIENT SYSTEMS

PART-2

LTI Cont. Time Signal



$$\boxed{y(t) = x(t) \otimes h(t)}$$

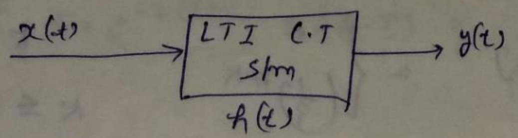
$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

where

$$x(\tau) = x(t)\Big|_{t \to \tau}$$

$$h(t-\tau) = h(t)\Big|_{t \to t-\tau}$$

Ques



$x(t) = e^{-2t}\, u(t)$

$h(t) = 5\, u(t)$

$y(t) = ?$

Sol^n:- $y(t) = x(t) \otimes h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

$x(t) = e^{2t} u t$

$$x(\tau) = e^{-2\tau} u(\tau) = \begin{cases} 0 & \tau < 0 \\ e^{-2t} & \tau \geq 0 \end{cases}$$

$h(t) = 5u(t)$

$$h(t-\tau) = 5u(t-\tau) = \begin{cases} 0 & \tau > t \\ 5 & \tau \leq t \end{cases}$$

$$y(t) = \int_0^t e^{-2\tau} \cdot 5\, d\tau = 5 \int_0^t e^{-2\tau}\, d\tau = 5\left[\frac{e^{-2\tau}}{-2}\right]_0^t$$

$$= -\frac{5}{2}\left[e^{-2t} - e^{0}\right] \Rightarrow -\frac{5}{2} e^{-2t} + \frac{5}{2}$$

$$y(t) = \frac{5}{2}\left[1 - e^{-2t}\right] \quad \text{Ans}$$

Ques:- $x(t) = 2 \cdot e^{-3t} \, u(t)$

$h(t) = 5 \cdot e^{-2t} \, u(t)$

$y(t) = ?$

Sol^n:-

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \, h(t-\tau) \, d\tau$$

$x(t) = 2e^{-3t} \, u(t)$

$$x(\tau) = 2e^{-3\tau} \, u(\tau) = \begin{cases} 0 & \tau < 0 \\ 2e^{-3\tau} & \tau \geq 0 \end{cases}$$

$h(t) = 5e^{-2t} \, u(t)$

$$h(t-\tau) = 5 \cdot e^{-2(t-\tau)} \, u(t-\tau) = \begin{cases} 0 & \tau > t \\ 5 \cdot e^{-2(t-\tau)} & \tau \leq t \end{cases}$$

$$y(t) = \int_0^t 2 \cdot e^{-3\tau} \cdot 5 \, e^{-2(t-\tau)} d\tau$$

$$= 10 \int_0^t e^{-3\tau} \cdot e^{-2t} \cdot e^{2\tau} \cdot d\tau$$

$$= 10 \cdot e^{-2t} \int_0^t e^{-\tau} d\tau$$

$$= 10 e^{-2t} \left[\frac{e^{-\tau}}{-1} \right]_0^t$$

$$= -10 \cdot e^{-2t} (e^{-t} - 1)$$

$$= 10 \, e^{-2t} (1 - e^{-t}) \quad \text{Ans}$$

Ques:- $x(t) = 5 \, u(t-3)$

$h(t) = 2 \cdot e^{-2t} u(t)$

$y(t) = ?$

Sol^n:-

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t) = 5\,u(t-3)$

$$x(\tau) = 5\,u(\tau-3) = \begin{cases} 0 & \tau < 3 \\ 5 & \tau \geq 3 \end{cases}$$

$h(t) = 2\,e^{-2t}\,u(t)$

$$h(t-\tau) = 2 \cdot e^{-2(t-\tau)}\, u(t-\tau) = \begin{cases} 0 & \tau > t \\ 2\,e^{-2(t-\tau)} & \tau \leq t \end{cases}$$

$$y(t) = \int_3^t 5 \cdot 2 \cdot e^{-2(t-\tau)}\, d\tau$$

$$= 10 \int_3^t e^{-2t} \cdot e^{2\tau}\, d\tau$$

$$= 10 e^{-2t} \int_3^t e^{2\tau}\, d\tau$$

$$= 10 e^{-2t} \left[\frac{e^{2\tau}}{2} \right]_3^t$$

$$= \frac{10 e^{-2t}}{2} \left[e^{2t} - e^{6} \right]$$

$$= 5 \left[1 - e^{6} \cdot e^{-2t} \right]$$

$$y(t) = 5 \left[1 - e^{-2(t-3)} \right] \quad \text{Ans}$$

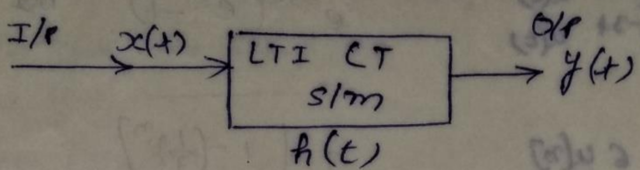
Properties of LTI s/m:-

(1) Commutative Property.
(2) Distributive Property.
(3) Associative Property.

(1) For LTI Cont. Time s/m

(i) Commutative Property

Case (i)



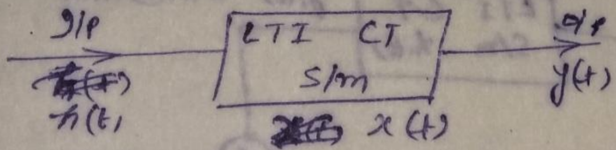
O/P of the s/m

$$\boxed{y(t) = x(t) \otimes h(t)}$$

Or

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau}$$

Case (ii) If I/P and Transfer fn are Interchanged



O/P remains same

$$\boxed{y(t) = h(t) \otimes x(t)}$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau}$$

According to commutative Property

$$\boxed{y(t) = x(t) \otimes h(t) = h(t) \otimes x(t)}$$

Mathematically $\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) \cdot x(t-\tau) d\tau}$

For discrete time S/m

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=-\infty}^{\infty} h(k).\, x(n-k)}$$

Or

$$\boxed{y[n] = x[n] \otimes h[n] = h[n] \otimes x[n]}$$

Ques

$x(t) = e^{-2t} u(t)$

$h(t) = e^{-3t} u(t)$

$y(t) = ?$

Ans $e^{-3t}(e^{t} - 1)$

Ques

$x[n] = 6\, u[n]$

$h[n] = \left(\frac{1}{3}\right)^n u[n]$

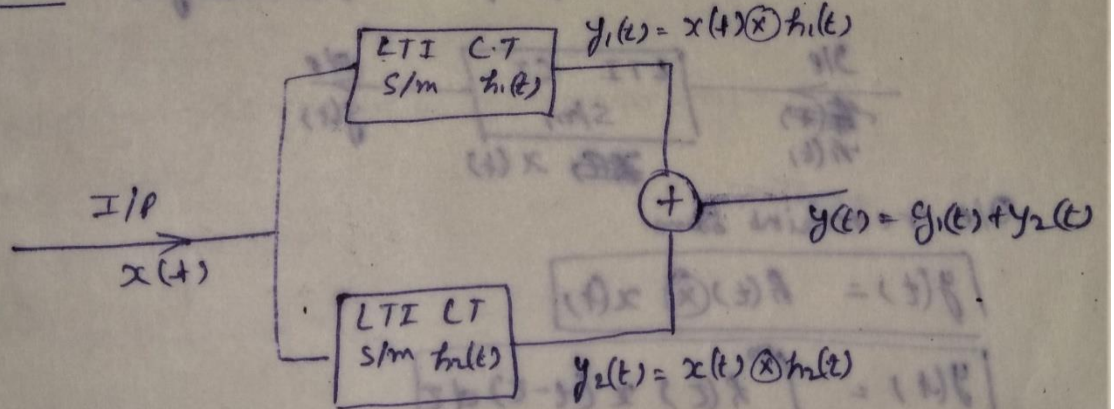
$y[n] = ?$

$9\left[1 - \left(\frac{1}{3}\right)^n\right]$

Property 2:- Distributive property.

For Cont. Time S/m

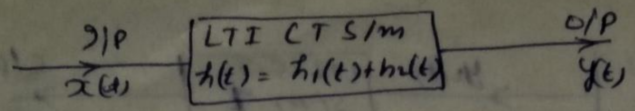
Case (1)



$\therefore y(t) = y_1(t) + y_2(t)$

Or $y(t) = x(t) \otimes h_1(t) + x(t) \otimes h_2(t)$

Case II



$y(t) = x(t) \otimes h(t)$

$y(t) = x(t) \otimes \{h_1(t) + h_2(t)\}$

Ques - $x(t) = e^{-3t} u(t)$
$h(t) = 5u(t) - 5u(t-3)$

In both cases O/P remains same

According to distributive property

$y(t) = x(t) \otimes \{h_1(t) + h_2(t)\}$

$$\boxed{y(t) = x(t) \otimes h_1(t) + x(t) \otimes h_2(t)}$$

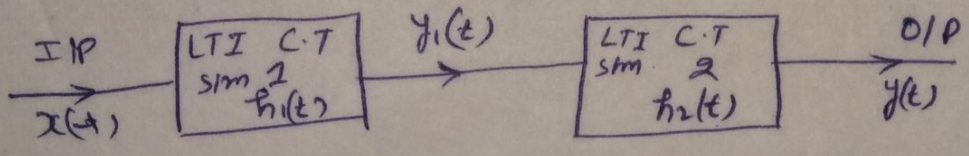
Similarly

$$\boxed{y(t) = x(t) \otimes \{h_1(t) - h_2(t)\} = x(t) \otimes h_1(t) - x(t) \otimes h_2(t)}$$

Property 3 :- Associative Property

For Cont. Time S/m

Case I

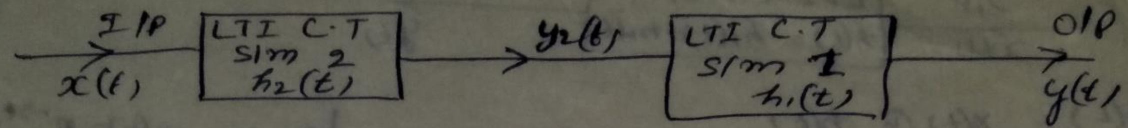


$y_1(t) = x(t) \otimes h_1(t)$

$y(t) = y_1(t) \otimes h_2(t)$

$$\boxed{y(t) = \{x(t) \otimes h_1(t)\} \otimes h_2(t)}$$

Case ii



$$y_2(t) = x(t) \circledast h_2(t)$$

$$y(t) = y_2(t) \circledast h_1(t)$$

$$\boxed{y(t) = \{x(t) \circledast h_2(t)\} \circledast h_1(t)}$$

In both cases o/p remains same According to Associative Property

$$\boxed{y(t) = \{x(t) \circledast h_1(t)\} \circledast h_2(t) = \{x(t) \circledast h_2(t)\} \circledast h_1(t)}$$