Class - B.tech 2nd yr ECE
Sub - Signal & System
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Topic - Nyquist Rate & Nyquist Interval

* It is the minimum Sampling time required to represent the continuous signal x(t) " faithfully in its sampled form"

* According to Nyquist Sampling theorem the Nyquist rate is $f_{s,min} = 2f_m$. If the sampling frequency f_s is less than the Nyquist rate then a distortion

called "aliasing" is introduced in the spectrum of the sampled signal.

∴ Nyquist criteria:

$$f_{s(min)} = 2f_m \text{ samples/sec}$$

or $$T_{s(max)} = \frac{1}{f_{s(min)}} = \frac{1}{2f_m} \text{ sec}$$

* $T_{s(max)}$ is called as the Nyquist interval.
* So, in short, it means that the signal must be sampled atleast twice during each cycle of its highest frequency component.

Numerical

Q1. The continuous time signal is represented by the following eq.

$x(t) = (4\sin 6283t + 5\sin 1256t)$

is to be sampled. calculate the minimum sampling frequency $f_s(min)$.

Sol:- The signal $x(t)$ consists of two sinusoidal components.

* The frequency of the first sinusoidal component =

$$f_1 = \frac{6283}{2\pi} = 1\,kHz.$$

The frequency of second sinusoidal component

$$f_2 = \frac{12566}{2\pi} = 2\,KHz.$$

Thus the highest frequency in the continuous time signal is $f_2 = 2\,KHz$.

Hence the minimum sampling frequency.

$$f_{s(min)} = 2 \times f_2.$$

$$\therefore f_{s(min)} = 2 \times 2\,kHz$$

$$= 4\,KHz.$$

Q.2 Find the Nyquist rate and Nyquist Interval for each of the following signals.

(a) $x(t) = 5 \cos 1000\pi t \cos 4000\pi t$.

(b) $x(t) = \dfrac{\sin 200\pi t}{\pi t}$.

Sol:- The given signal $x(t)$ is in the form of product of cosine terms so let us the following standard. So let us use the following standard expression:

$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

$\therefore 5\cos 1000\pi t \cos 4000\pi t$

$= 2.5\cos 5000\pi t + 2.5\cos 3000\pi t$

$x(t) = 2.5\cos 5000\pi t + 2.5\cos 3000\pi t$ ---(1)

From the equation(1) it is clear that the maximum frequency component present in the signal $x(t)$ is of 2500 Hz. In other words $x(t)$ is ~~Bandwidth~~ limited to 2.5 KHz ($W = 2.5$ KHz).

i. Nyquist rate $= 2W$

$= 2 \times 2.5$ kHz $= 5$ KHz

Nyquist Interval

$= \frac{1}{2W} = \frac{1}{5 \times 10^3} = 0.2$ msec.

(b) $x(t) = \frac{\sin 200\pi t}{\pi t}$

$x(t) = \frac{1}{\pi t} \sin(2\pi 100t)$

* In order to calculate the Nyquist rate we need to calculate the maximum frequency component present in its spectrum

* freq of given signal is 100 Hz

∴ Nyquist rate is 200 Hz and the Nyquist interval is $\left(\frac{1}{200}\right)$

i.e. 5 msec.