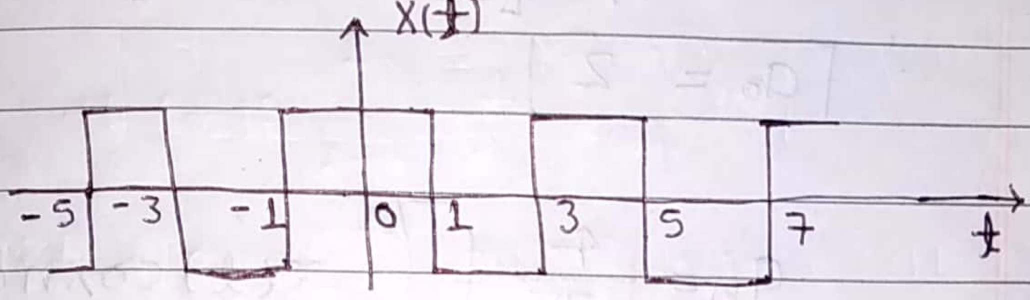
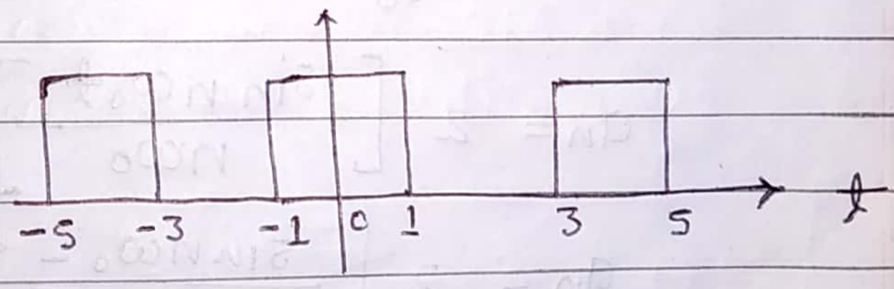
Q. Find the trigonometric Fourier series for the periodic signal shown in fig.



Sol^n Assumption:— Shift the signal upward by 1 unit



$$x(t) = \begin{cases} 2 ; & -1 < t < 1 \\ 0 ; & 1 < t < 3 \end{cases}$$

Fundamental time period = 4

$$\omega_0 = \frac{2\pi}{T}$$

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_0 = \frac{4}{T}\int_0^{T/2} x(t)\,dt$$

$$a_0 = \frac{4}{4}\left[\int_0^1 2\,dt + \int_1^2 0\,dt\right]$$

$$\boxed{a_0 = 2}$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega_0 t\,dt$$

$$a_n = \frac{4}{T}\left[\int_0^1 2\cos n\omega_0 t\,dt + \int_1^2 0\cos n\omega_0 t\,dt\right]$$

$$a_n = 2\int_0^1 \cos n\omega_0 t\,dt + 0$$

$$a_n = 2\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_0^t$$

$$a_n = 2\left[\frac{\sin n\omega_0 - \sin n\omega_0 \cdot 0}{n\omega_0}\right]$$

$$a_n = \frac{2\sin n\omega_0}{n\omega_0}$$

putting $\omega_0 = \frac{\pi}{2}$

$$a_n = \frac{2\sin\frac{n\pi}{2}}{\frac{n\pi}{2}} = \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

DATE: / 201

$$a_n = \begin{cases} 0 \;; & n \text{ even} \\ \dfrac{4(-1)^n}{n\pi} \;; & n \text{ odd} \end{cases}$$

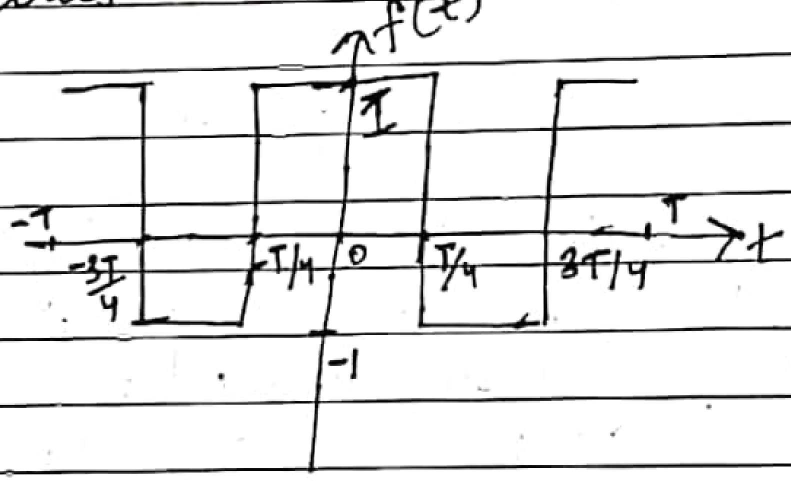
since, given signal is even,

so

$$b_n = 0$$

$$\boxed{x(t) = 1 + \sum_{n=\text{odd}}^{\infty} \frac{4(-1)^n}{n\pi} \cos n\frac{\pi}{2} t}$$

Que Expand square wave voltage signal as shown in fig. into a fourier series



$$f(t) = \begin{cases} 1, & 0 < t < T/4 \\ -1; & T/4 < t < 3T/4 \\ 1; & 3T/4 < t < T \end{cases}$$

Sol By inspecting the stare wave the average over one period is zero so that, $a_0 = 0$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt$$

$$\Rightarrow \frac{2}{T}\left[\int_0^{T/4} \cos n\omega t\, dt - \int_{T/4}^{3T/4} 1\cos n\omega t\, dt + \int_{3T/4}^{T} 1\cos n\omega t\, dt\right].$$

$$\Rightarrow \frac{2}{n\omega T}\left[\sin \frac{n\omega T}{4} - \left(\sin\frac{3n\omega T}{4} - \sin\frac{n\omega T}{4}\right) + \left(\sin n\omega T - \sin\frac{3n\omega T}{4}\right)\right]$$

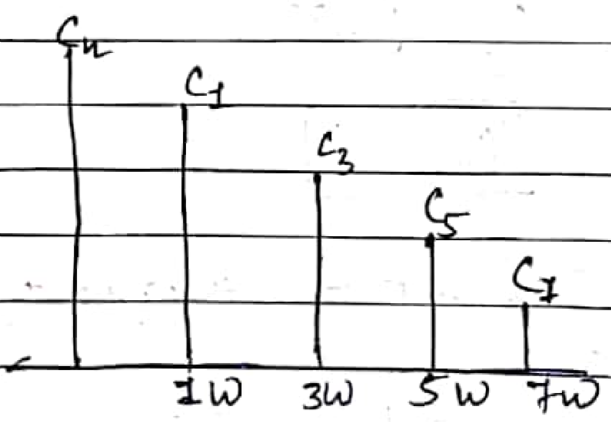
$$\Rightarrow \frac{2}{n\omega T}\left[2\sin\frac{n\omega T}{4} - 2\sin\frac{3n\omega T}{4} + \sin n\omega T\right]$$

$\Downarrow$ become 0 as

$\omega T = 2\pi$

$\sin n\omega T = 0$

$$\Rightarrow \frac{4}{n\omega T}\left(\sin\frac{n\omega T}{4} - \sin\frac{3n\omega T}{4}\right)$$



Amplitude Spectrum

for all integer value of n;

$$a_n = \frac{4}{n\pi} \; ; n = 1, 5, 9.$$

$$= \frac{-4}{n\pi} \; ; n = 3, 7, 11.$$

$$= 0 \quad n = \text{even}$$

$$= \frac{4}{2\pi}\left[\frac{\sin 2\pi}{4} - \frac{\sin 6\pi}{4}\right].$$

$$= \frac{\cancel{4}}{2\pi} \times \frac{1}{\cancel{4}}\left[\sin 2\pi - \sin 6\pi\right]$$

$$= \frac{1}{2\pi}\left[\sin 2\pi - \sin 6\pi\right]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t \, dt$$

$$b_n = 0$$

$$c_n = \sqrt{a_n^2 + b_n^2}$$

as $b_n = 0$.

$$\Rightarrow c_n = \sqrt{a_n^2 + 0}.$$

$$c_n = \sqrt{a_n^2}$$

$$\Rightarrow \boxed{c_n = a_n}$$

$$f(t) = \frac{4}{\pi}\left[\cos \omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t\right.$$

$$- \frac{1}{7}\cos 7\omega t + \text{———}$$

Q Let us find the exponential series for the following rectangular wave, given by

$$f(t) = 4, \quad 0 < t < 1$$
$$= -4, \quad 1 < t < 2$$
$$f(t+2) = f(t)$$

Soln

$T = 2$ (Time period)

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

(By Formula)

$$C_n = \frac{1}{2}\int_{-1}^{0}(-4)e^{-Jn\pi t}\,dt + \frac{1}{2}\int_{0}^{1} 4\,e^{-Jn\pi t}\,dt$$

$$C_n = \frac{4}{Jn\pi}\left[1-(-1)^n\right]$$

Also, we have

$$C_n = \frac{1}{2}\int_{-1}^{1} f(t)\,dt$$

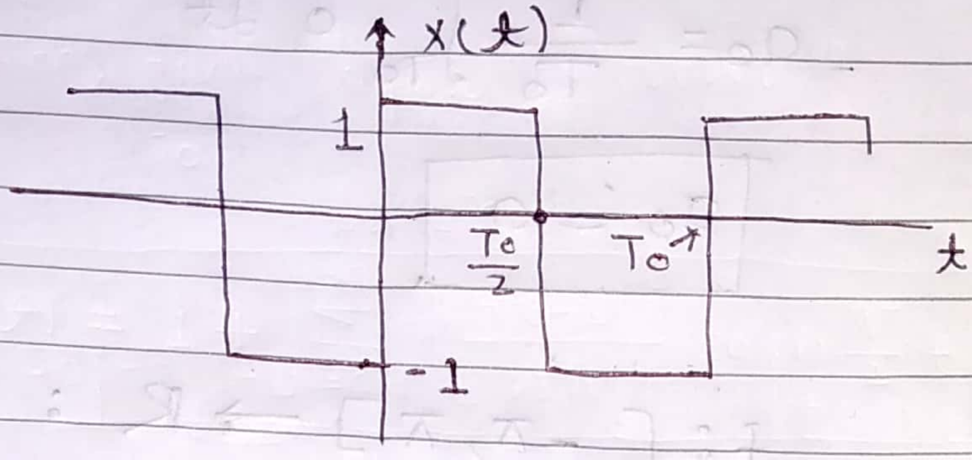
$$C_n = \frac{1}{2}\int_{-1}^{0} 4\,dt - \frac{1}{2}\int_{0}^{1} 4\,dt$$

$$C_n = 0$$

Since $C_n = 0$ for n even & $C_n = 8/Jn\pi$ for n odd, we may write the exponential series in the form

$$\boxed{f(t) = \frac{8}{J\pi}\sum_{n=-\infty}^{\infty}\frac{1}{2n-1}e^{J(2n-1)\pi t}}$$

Q Determination of the fourier series cofficients for an antisymmetric period square wave.



Sol^n:

$$x(t) = \begin{cases} -1 & ; -\frac{T_0}{2} < t < 0 \\ +1 & ; 0 < t < \frac{T_0}{2} \end{cases}$$

$$a_n = \frac{1}{T_0} \int_{-T_0/2}^{0} (-1) e^{-jn\omega_0 t} dt + \frac{1}{T_0} \int_{0}^{T_0/2} (1) e^{-j\omega_0 n t} dt$$

$$a_n = \frac{1}{T_0} \left[-\left\{ \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right\}_{-T_0/2}^{0} + \left\{ \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right\}_{0}^{T_0/2} \right]$$

$$\boxed{a_n = \frac{1}{j\pi n} \left\{ 1 - (-1)^n \right\}} \quad n \neq 0$$

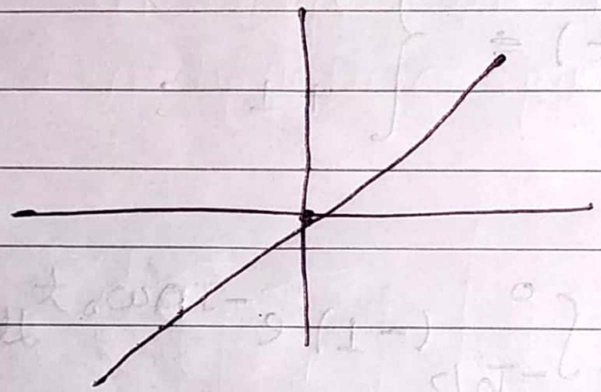
PAGE No
DATE : / 201

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$a_0 = \frac{1}{T_0} \int_{T_0} 0 \, dt$$

$$\boxed{a_0 = 0}$$

Q $f : [-\pi, \pi] \to \mathbb{R} : x \mapsto x$



$f(x) = x$ for odd

Solⁿ The graph is odd, so

$a_n = 0$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x \sin nx \, dx$$

$$b_n = \frac{2}{\pi}\left[\left[-\frac{x\cos nx}{n}\right]_0^{\pi} + \frac{1}{n}\int_0^{\pi} 1 \cdot \cos nx \, dx\right]$$

$$= \frac{2}{\pi}\left[-\frac{\pi \cos n\pi}{n} + \underbrace{\left[\frac{\sin nx}{n^2}\right]_0^{\pi}}_{=0}\right]$$

$$= -\frac{2(-1)^n}{n}$$

$$\boxed{\text{Fourier series} = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx}$$