Q.1 Find the Fourier transform of the following function using the properties of fourier transform.

$$y(t) = \frac{d}{dt} t e^{-3t} u(t) \otimes e^{-2t} u(t)$$

Sol^n The function y(t) can be written as,

$$y(t) = \underbrace{\frac{d}{dt} t e^{-3t} u(t)}_{x_1(t)} \otimes \underbrace{e^{-2t} u(t)}_{x_2(t)}$$

$$y(t) = x_1(t) \otimes x_2(t)$$

Now, we can use Convolution propety of fourier transform,

$$Y(\omega) = X_1(\omega) \cdot X_2(\omega) \quad \text{——— } ①$$

$$x_1(t) = \frac{d}{dt} t e^{-3t} u(t)$$

$$F[e^{-3t} u(t)] = \frac{1}{3 + j\omega}$$

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using the FT property of differentiation in frequency, we get

$$F[te^{-3t}u(t)] = \frac{jd}{d\omega}\left[\frac{1}{3+j\omega}\right] = \frac{1}{(3+j\omega)^2} \quad \text{--- (2)}$$

Now using the FT property of differentiation in time, we get

$$F\left[\frac{d}{dt}\, t e^{-3t}u(t)\right] = F[x_1(t)] = j\omega F[te^{-3t}u(t)]$$

$$= \frac{j\omega}{(3+j\omega)^2} \quad \text{--- (3)}$$

& $x_2(t) = e^{-2t}u(t)$

$$X_2(\omega) = \frac{1}{2+j\omega} \quad \text{--- (4)}$$

$$Y(\omega) = X_1(\omega) \cdot X_2(\omega)$$

$$Y(\omega) = \frac{j\omega}{(3+j\omega)^2} \cdot \frac{1}{(2+j\omega)}$$

$$\boxed{Y(\omega) = \frac{j\omega}{(3+j\omega)^2(2+j\omega)}}$$

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② A signal, $x(t)$ has a Fourier transform given by $X(\omega) = \frac{1}{(1+\omega^2)}$. write down the Fourier transform of $x\left(\frac{3t}{2} - 1\right)$.

Solution :-

$$F[x(t)] = X(\omega) = \frac{1}{1+\omega^2} \quad \text{——} \quad ①$$

By using Time shifting & Time scaling property

$$F[x(t-1)] = e^{-j\omega} X(\omega)$$

$$F[x(t-a)] = e^{-j\omega a} X(\omega)$$

$$F\left[x\left[\frac{3}{2}t - 1\right]\right] = \frac{1}{(3/2)} e^{-(j\omega/3/2)} X\left(\frac{\omega}{3/2}\right)$$

$$\left\{ F[x(at)] = \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \right\}$$

Thus,

$$F\left[x\left(\frac{3}{2}t - 1\right)\right] = \frac{2}{3} e^{\frac{-j2\omega}{3}} \frac{1}{1+\left(\frac{2\omega}{3}\right)^2}$$

$$= \frac{2}{3} e^{\frac{-2j\omega}{3}} \frac{9}{9+4\omega^2}$$

$$\boxed{F\left[x\left(\frac{3}{2}t - 1\right)\right] = \frac{6\, e^{-2j\omega/\omega}}{9+4\omega^2}}$$

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Q.3 Determine the Fourier transform of the rectangular

$$x(t) = rect\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \text{for } -\frac{\tau}{2} < t < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find the continuous-time Fourier transform of the rectangular signal. Also plot its magnitude response.

Solⁿ The Fourier transform is given by,

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-J\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} rect\left(\frac{t}{\tau}\right) e^{-J\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-J\omega t}\, dt$$

$$X(\omega) = \left[\frac{e^{-J\omega t}}{-J\omega}\right]_{-\tau/2}^{\tau/2} = \left[\frac{e^{-J\omega\tau/2} - e^{+J\omega\tau/2}}{-J\omega}\right]$$

$$= \left[\frac{-2j\sin\left(\frac{\omega\tau}{2}\right)}{-J\omega}\right]$$

$$= \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\omega/2}$$

$$= \tau \cdot \frac{\sin\left(\frac{\omega\tau}{2}\right)}{(\omega\tau/2)}$$

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$$\boxed{X(\omega) = \tau \, \text{sinc}(\omega\tau/2)}$$

Since, $\text{sinc}(x) = 0$ for $x = \pm n\pi$

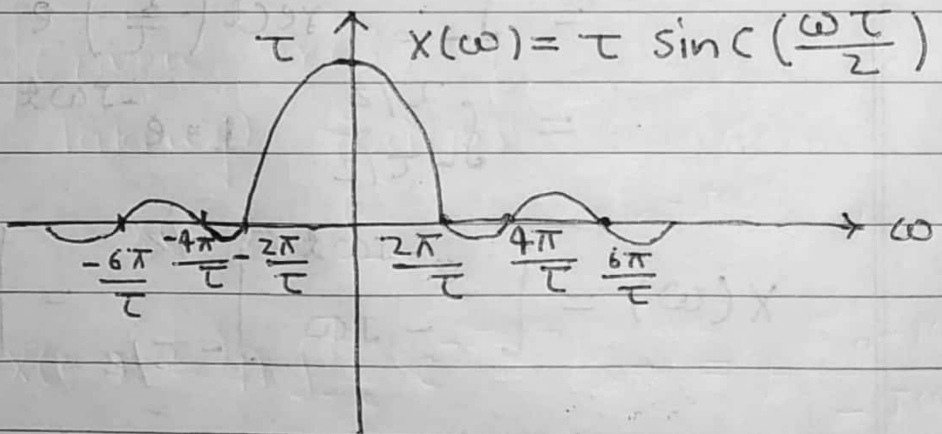
and $\text{sinc}(0) = 1$

Therefore, $\text{sinc}(\omega\tau/2) = 0$ for $(\omega\tau/2) = \pm n\pi$

or $\omega = \dfrac{\pm 2n\pi}{\tau}$ $n = 1, 2, 3$

$$\omega = \pm\frac{2\pi}{\tau}, \pm\frac{4\pi}{\tau}, \pm\frac{6\pi}{\tau}, \pm\frac{8\pi}{\tau}, \ldots$$

The plot of the $X(\omega)$ is given in fig.



Q.4 Using Fourier transform, find the convolution of :

$$x_1(t) = e^{-2t} \cdot u(t)$$

$$x_2(t) = e^{-3t} \cdot u(t)$$

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<u>solution-</u> $x_1(t) = e^{-2t}\, u(t)$

$$X_1(\omega) = \frac{1}{j\omega + 2} \quad \text{--- (1)}$$

&

$$x_2(t) = e^{-3t}\, u(t)$$

$$X_2(\omega) = \frac{1}{j\omega + 3} \quad \text{--- (2)}$$

Using convolution property of fourier transform

$$x_1(t) \circledast x_2(t) = F^{-1}\left[X_1(\omega) \cdot X_2(\omega)\right]$$

$$= F^{-1}\left[\frac{1}{(j\omega+2)(j\omega+3)}\right]$$

--- (3)

Now, we can use partial fraction

on $\frac{1}{(j\omega+2)(j\omega+3)} = \frac{1}{j\omega+2} - \frac{1}{j\omega+3}$

putting in eq^n 3^{rd} & get -

$$x_1(t) \circledast x_2(t) = F^{-1}\left[\frac{1}{j\omega+2} - \frac{1}{j\omega+3}\right]$$

$$\boxed{x_1(t) \circledast x_2(t) = e^{-2t}\, u(t) - e^{-3t}\, u(t)}$$

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Q.5 Explain Fourier transform of single sided exponential pulse.

OR

Find the Fourier transform of the signals given below:

(i) $x(t) = \begin{cases} A, & |t| < T_0 \\ 0, & |t| > T_0 \end{cases}$

(ii) $x(t) = e^{-at} \; u(t)$

& Draw also magnitude & phase response (plot).

Sol^n (i) Given $x(t) = \begin{cases} A, & |t| < T_0 \\ 0, & |t| > T_0 \end{cases}$

Taking Fourier transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = A \int_{-T_0}^{T_0} e^{-j\omega t}\, dt = A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T_0}^{T_0}$$

$$= \frac{-A}{j\omega}\left[e^{-j\omega T_0} - e^{j\omega T_0}\right]$$

$$= \frac{A}{j\omega}\left[e^{j\omega T_0} - e^{-j\omega T_0}\right]$$

$$X(\omega) = \frac{2A}{\omega}\left[\frac{e^{j\omega T_0} - e^{-j\omega T_0}}{2j}\right]$$

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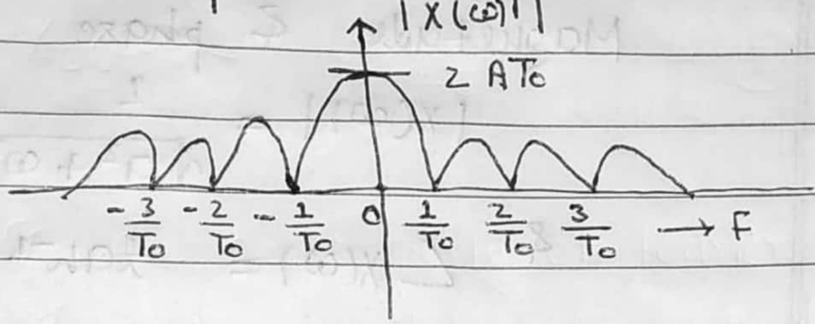
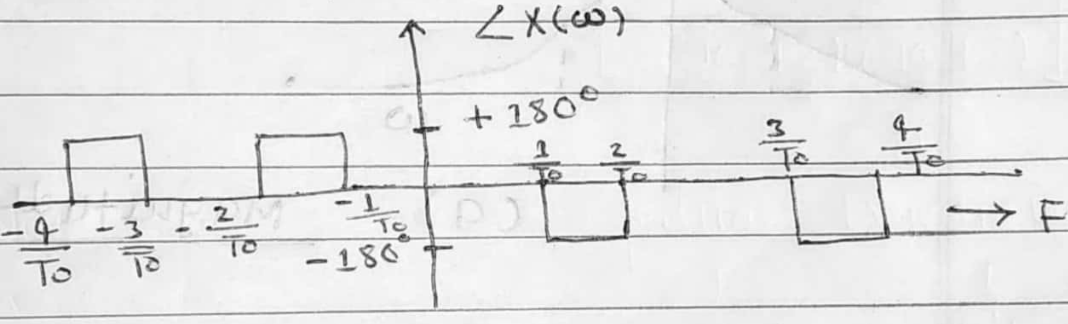
$$X(\omega) = \frac{2A}{\omega} \sin \omega T_0$$

$$= 2AT_0 \frac{\sin \omega T_0}{\omega T_0} = 2AT_0 \operatorname{sinc} \omega T_0$$

Magnitude & phase response of $X(\omega)$:



phase spectrum



(ii) $x(t) = e^{-at} u(t)$

$$x(t) = \begin{cases} e^{-at}, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

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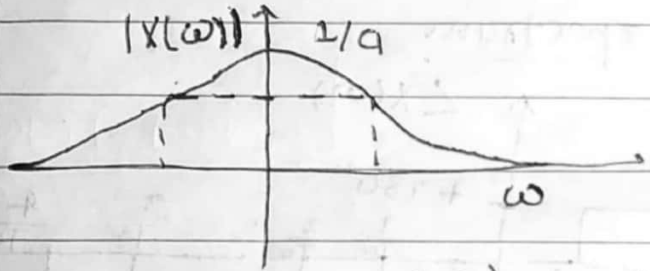
$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^\infty$$

$$= \frac{-1}{(a+j\omega)} [0-1] = \frac{1}{a+j\omega}$$

Magnitude & phase response:

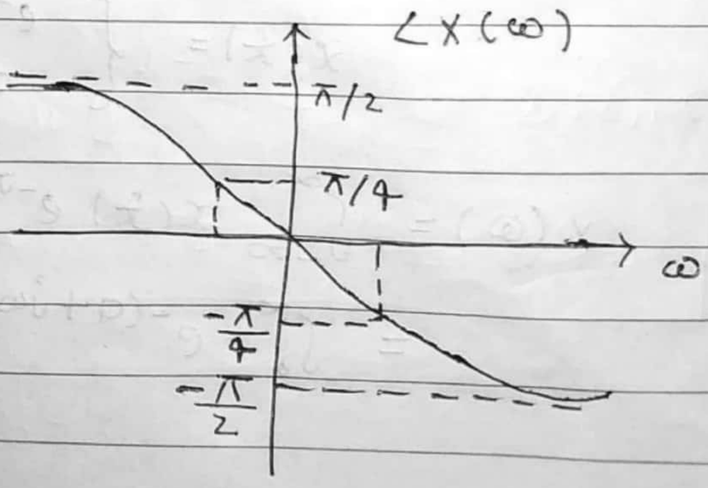
$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\& \quad \angle X(\omega) = \tan^{-1}\left(\frac{\omega}{a}\right)$$



(a) Magnitude spectrum

(b) Phase spectrum



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