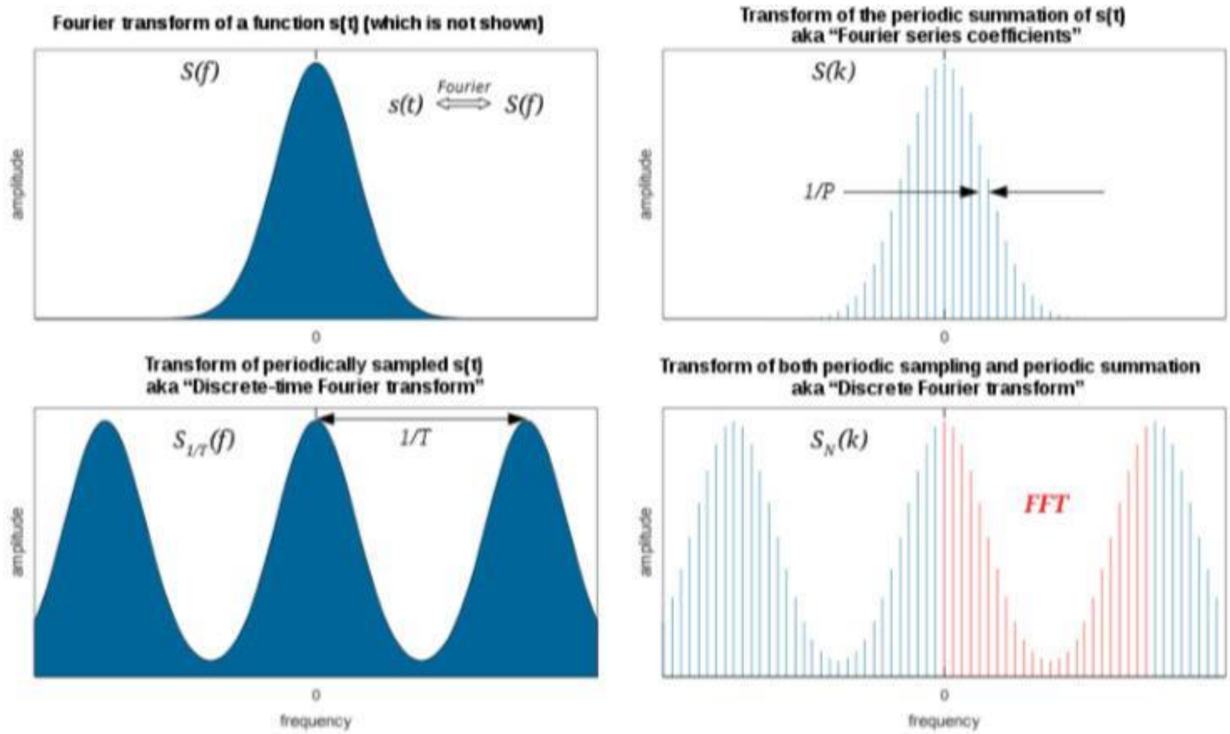
Difference between DFT and DTFT

Discrete Fourier transform

In mathematics, the discrete Fourier transform **(DFT) converts a finite sequence of equally**-spaced samples of a function into a same-length sequence of equally-spaced samples of the discrete time Fourier transform (DTFT), which is a complex-valued function of frequency. The interval at which the DTFT is sampled is the reciprocal of the duration of the input sequence. An inverse DFT is a Fourier **series, using the DTFT samples as coefficients of** complex sinusoids at the corresponding DTFT frequencies. It has the same sample-values as the original input sequence. The DFT is therefore said to be a frequency domain representation of the original input sequence. If the

original sequence spans all the non-zero values of a function, its DTFT is continuous (and periodic), and the DFT provides discrete samples of one cycle. If the original sequence is one cycle of a periodic function, the DFT provides all the non-zero values of one DTFT cycle.



Depiction of a Fourier transform (upper left) and its periodic summation (DTFT) in the lower left corner. The spectral sequences at (a) upper right and (b) lower right are respectively computed from (a) one cycle of the periodic summation of s(t) and (b) one cycle of the periodic summation of the s(nT) sequence. The respective formulas are (a) the Fourier series integral and (b) the DFT summation. Its similarities to the original transform, S(f), and its relative computational ease are often the motivation for computing a DFT sequence.

The DFT is the most important discrete transform, used to perform Fourier analysis in many practical applications.[1] In digital signal processing, the function is any quantity or signal that varies over time, such as the pressure of a sound wave, a radio signal, or daily temperature readings, sampled over **a finite time interval (often defined by a window** function[2]). In image processing, the samples can be the values of pixels along a row or column of a **raster image. The DFT is also used to efficiently** solve partial differential equations, and to perform other operations such as convolutions or multiplying large integers. **Since it deals with a finite** amount of data, it can be implemented in computers by numerical algorithms or even dedicated hardware. These implementations usually **employ efficient fast Fourier transform** (FFT) algorithms;[3] so much so that the terms "FFT" and "DFT" are often used interchangeably. Prior to its current usage, the "FFT" initialism may have also **been used for the ambiguous term "finite Fourier** transform".

Discrete-time Fourier transform

In mathematics, the discrete-time Fourier transform (DTFT) is a form of Fourier analysis that is applicable to a sequence of values.

The DTFT is often used to analyze samples of a continuous function.

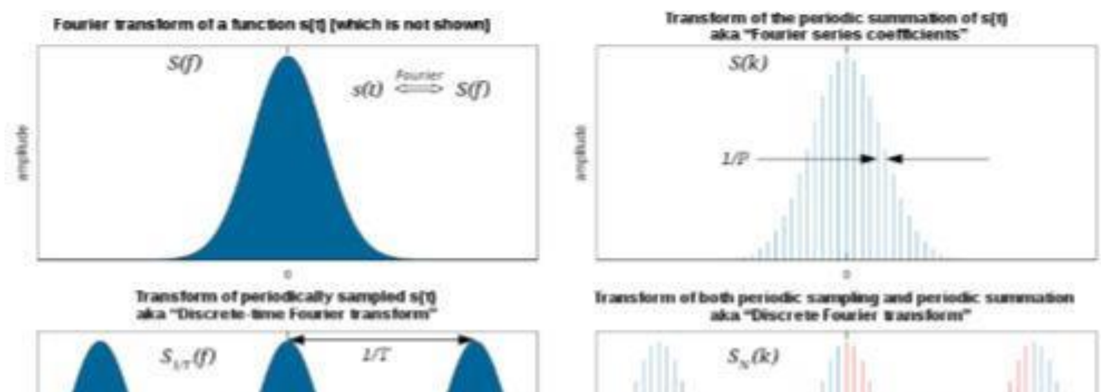
The Fourier transforms Continuous Fourier transform Fourier series Discrete-time Fourier transform Discrete Fourier transform Discrete Fourier transform over a ring Fourier analysis Related transforms term discrete-time refers to the fact that the transform operates on discrete data, often samples whose interval has units of time. From uniformly spaced samples it produces a function of frequency that is a periodic summation of the continuous Fourier transform of the original continuous function. Under certain theoretical conditions, described by the sampling theorem, the original continuous function can be recovered perfectly from the DTFT and thus from the original discrete samples. The DTFT itself is a continuous function of frequency, but discrete samples of it can be readily calculated via the discrete Fourier transform (DFT) (see § Sampling the DTFT), which is by far the most common method of modern Fourier analysis.

Both transforms are invertible. The inverse DTFT is the original sampled data sequence. The inverse DFT is a periodic summation of the original sequence. The fast Fourier transform (FFT) is an algorithm for computing one cycle of the DFT, and its inverse produces one cycle of the inverse DFT.

The discrete-time Fourier transform of a discrete set of real or complex numbers x[n], for all integers n, is a Fourier series, which produces a periodic function of a frequency variable. When the **frequency variable, ω, has normalized units of**

D**efinition radians/sample, the periodicity is 2π, and** the Fourier series is:[1]

The utility of this frequency domain function is rooted in the Poisson summation formula. Let X(f) be the Fourier transform of any function, x(t), whose samples at some interval T (seconds) are equal· (or proportional) to the x[n] sequence, i.e. T x(nT) = x[n]. Then the periodic function represented by the Fourier series is a periodic summation of X(f) in terms of frequency f in hertz (cycles/sec):[a]



Depiction of a Fourier transform (upper left) and its periodic summation (DTFT) in the lower left corner.

The lower right corner depicts samples of the DTFT that are computed by a discrete Fourier transform (DFT).

The integer k has units of cycles/sample, and 1/T is the sample-rate, fs (samples/sec). So X1/T(f) comprises exact copies of X(f) that are shifted by

multiples of fs hertz and combined by addition. For **sufficiently large fs the k = 0 term can be observed in the region [−fs/2, fs/2] with**

Fig 1. Depiction of a Fourier transform (upper left) and its periodic summation (DTFT) in the lower left corner. The lower right corner depicts samples of the DTFT that are computed by a discrete Fourier transform (DFT).

little or no distortion (aliasing) from the other terms. In Fig.1, the extremities of the distribution in the upper left corner are masked by aliasing in the periodic summation (lower left).

We also note that e−i2πfTn is the Fourier transform of δ(t − nT).