1850 MHz.

4.2 Impulse Response Model of a Multipath Channel

The small-scale variations of a mobile radio signal can be directly related to the impulse response of the mobile radio channel. The impulse response is a wideband channel characterization and contains all information necessary to simulate or analyze any type of radio transmission through the channel. This stems from the fact that a mobile radio channel may be modeled as a linear filter with a time varying impulse response, where the time variation is due to receiver motion in space. The filtering nature of the channel is caused by the summation of amplitudes and delays of the multiple arriving waves at any instant of time. The impulse response is a useful characterization of the channel, since it may be used to predict and compare the performance of many different mobile communication systems and transmission bandwidths for a particular mobile channel condition.

To show that a mobile radio channel may be modeled as a linear filter with a time varying impulse response, consider the case where time variation is due strictly to receiver motion in space. This is shown in Figure 4.2.

Figure 4.2
The mobile radio channel as a function of time and space.

In Figure 4.2, the receiver moves along the ground at some constant velocity v. For a fixed position d, the channel between the transmitter and the receiver can be modeled as a linear time invariant system. However, due to the different multipath waves which have propagation delays which vary over different spatial locations of the receiver, the impulse response of the linear time invariant channel should be a function of the position of the receiver. That is, the channel impulse response can be expressed as $h(d,t)$. Let $x(t)$ represent the transmitted signal, then the received signal $y(d,t)$ at position d can be expressed as a convolution of $x(t)$ with $h(d,t)$.

$$y(d,t) = x(t) \otimes h(d,t) = \int_{-\infty}^{\infty} x(\tau)h(d,t-\tau)d\tau \tag{4.3}$$

For a causal system, $h(d,t) = 0$ for $t<0$, thus equation (4.3) reduces to

$$y(d,t) = \int_{-\infty}^{t} x(\tau)h(d,t-\tau)d\tau \tag{4.4}$$

Since the receiver moves along the ground at a constant velocity v, the position of the receiver can by expressed as

$$d = vt \tag{4.5}$$

Substituting (4.5) in (4.4), we obtain

$$y(vt,t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau \tag{4.6}$$

Since v is a constant, $y(vt,t)$ is just a function of t. Therefore, equation (4.6) can be expressed as

$$y(t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau = x(t) \otimes h(vt,t) = x(t) \otimes h(d,t) \tag{4.7}$$

From equation (4.7) it is clear that the mobile radio channel can be modeled as a linear time varying channel, where the channel changes with time and distance.

Since v may be assumed constant over a short time (or distance) interval, we may let $x(t)$ represent the transmitted bandpass waveform, $y(t)$ the received waveform, and $h(t,\tau)$ the impulse response of the time varying multipath radio channel. The impulse response $h(t,\tau)$ completely characterizes the channel and is a function of both t and τ. The variable t represents the time variations due to motion, whereas τ represents the channel multipath delay for a fixed value of t. One may think of τ as being a vernier adjustment of time. The received signal $y(t)$ can be expressed as a convolution of the transmitted signal $x(t)$ with the channel impulse response (see Figure 4.3a).

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t,\tau)d\tau = x(t) \otimes h(t,\tau) \tag{4.8}$$

If the multipath channel is assumed to be a bandlimited bandpass channel, which is reasonable, then $h(t,\tau)$ may be equivalently described by a complex baseband impulse response $h_b(t,\tau)$ with the input and output being the complex envelope representations of the transmitted and received signals, respectively (see Figure 4.3b). That is,

$$r(t) = c(t) \otimes \frac{1}{2}h_b(t,\tau) \tag{4.9}$$

4.2.1 Relationship Between Bandwidth and Received Power

In actual wireless communication systems, the impulse response of a multipath channel is measured in the field using channel sounding techniques. We now consider two extreme channel sounding cases as a means of demonstrating how the small-scale fading behaves quite differently for two signals with different bandwidths in the identical multipath channel.

Consider a pulsed, transmitted RF signal of the form

$$x(t) = \text{Re}\{p(t)\exp(j2\pi f_c t)\}$$

where $p(t)$ is a repetitive baseband pulse train with very narrow pulse width T_{bb} and repetition period T_{REP} which is much greater than the maximum measured excess delay τ_{max} in the channel. Now let

$$p(t) = 2\sqrt{\tau_{max}/T_{bb}} \quad \text{for } 0 \le t \le T_{bb}$$

and let $p(t)$ be zero elsewhere for all excess delays of interest. The low pass channel output $r(t)$ closely approximates the impulse response $h_b(t)$ and is given by

$$\begin{aligned} r(t) &= \frac{1}{2}\sum_{i=0}^{N-1} a_i(\exp(-j\theta_i)) \cdot p(t-\tau_i) \\ &= \sum_{i=0}^{N-1} a_i \exp(-j\theta_i) \cdot \sqrt{\frac{\tau_{max}}{T_{bb}}}\ \text{rect}\left[t - \frac{T_{bb}}{2} - \tau_i\right] \end{aligned} \tag{4.16}$$

To determine the received power at some time t_0, the power $|r(t_0)|^2$ is measured. The quantity $|r(t_0)|^2$ is called the *instantaneous multipath power delay profile* of the channel, and is equal to the energy received over the time duration of the multipath delay divided by τ_{max}. That is, using equation (4.16)

$$|r(t_0)|^2 = \frac{1}{\tau_{max}}\int^{\tau_{max}} r(t) \times r^*(t)\,dt \tag{4.17}$$