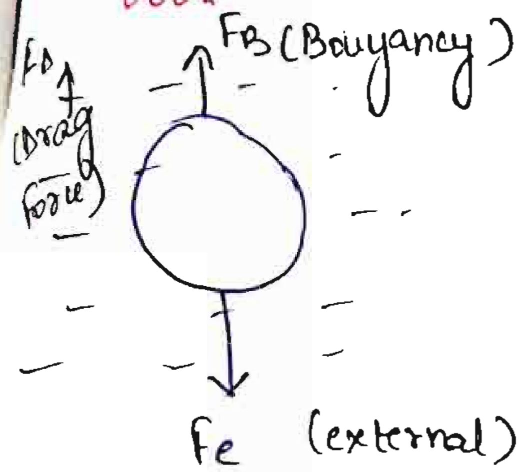
Mechanism of Particle Movement Through in A Fluid :-

Consider a spherical particle falling in fluid. Forces acting on this particle are :-



(i) F_e (external force) in ↓ direction

(ii) F_B (Bouyancy force) in ↑ direction

(iii) F_D (Drag force) in ↑ direction

At equilibrium :- $F = F_e - F_B - F_D$.

$$F = m \cdot \frac{dv}{dt}.$$

$$\therefore m \cdot \frac{dv}{dt} = m a_e - \frac{m \rho a_e}{\rho_p} - \frac{C_D v^2 A \rho}{2}.$$

$$\Rightarrow \boxed{\frac{dv}{dt} = a_e \left[1 - \frac{\rho}{\rho_p}\right] - \frac{C_D v^2 A \rho}{2m}} \quad — (A)$$

General eqn for the force acting on the body in any force field.

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Case 1: external force = gravity force i.e. $a_e = g$

$$\frac{dv}{dt} = g\left[1 - \frac{\rho}{\rho_p}\right] - \frac{C_D v^2 A \rho}{2m}$$

Case 2: external force = centrifugal force i.e. $a_e = r\omega^2$

$$\Rightarrow \frac{dv}{dt} = r\omega^2\left[1 - \frac{\rho}{\rho_p}\right] - \frac{C_D v^2 A \rho}{2m}.$$

Here,

F = resultant force on any body.

$\frac{dv}{dt}$ = acc^n of the body.

m = mass of body.

F_e = external force

F_b = Bouyancy force.

F_D = Drag force.

ρ_p = density of the particle.

ρ = " " fluid.

a_e = acc^n of the particle.

C_d = Drag coefficient.

v = velocity of the particle.

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(12)

...nal Velocity :- Consider the particle falling in gravitational field in such a manner that ...ther particle which might be ent do not ...inder its fall.

...2 the particle falls its velocity ↑ and will ...tinue to ↑ untill the accelerating and ...esisting forces are equal, when this point is ...eached the particle velocity remains constt. during the remaining of its fall unless the balance of the forces is disturbed, the ultimate constt. velocity is called "terminal velocity"

Law ... ...

Put $\frac{dv}{dt} = 0$ in eq^n (A)

Here, $m = \rho V = \frac{\rho \pi D_p^3}{6}$

$$g\left(1-\frac{\rho}{\rho_p}\right) = \frac{C_D V_t^2 A \rho}{2m}$$

$$g\left(1-\frac{\rho}{\rho_p}\right) = \frac{C_D V_t^2 \frac{\pi}{4} D_p^2 \cdot \rho}{2 \times \frac{\pi}{6} D_p^3}$$

$$\boxed{V_t = \sqrt{\frac{4 D_p g}{3 C_D \cdot \rho_p}(\rho_p - \rho)}}$$

This is known as Newton's Eq^n use to calcula[te] terminal velocity of falling spherical parti[cle] in laminar, transition or turbulent flow, C_D is evaluated

for turbulent region:-

R (Reynold's no.) lies b/w 1000 to 20,000

Drag coefficient = constt.

i.e. $\boxed{C_D = .44}$

then,

terminal velocity can be given as:-

$$V_t = 1.74\left[\sqrt{\frac{g D_p (\rho_p - \rho)}{\rho_p}}\right]$$

or $\boxed{V_t \propto D_p^{0.5}}$ → turbulent flow.

An expression for the terminal velocity independent of drag coefficient (C_D) may be developed for particle in laminar flow, the resisting force due to fluid friction acting on a sphere when the relative motion produces laminar flow is given by Stoke's eq^n:-

$\boxed{F_D = 3\pi D_p \cdot \mu v}$ laminar

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Let's find out v_t. Settling (Mech. Oper.)

$$m \cdot \frac{dv}{dt} = F_e - F_B - F_D.$$

$$m \cdot g \quad , \quad F_D = 3\pi D_p \cdot \mu \cdot v.$$

$$F_B = \frac{m \rho g}{\rho_p}.$$

$$mg - \frac{m\rho g}{\rho_p} - 3\pi D_p \mu v = 0$$

$$g\left[1 - \frac{\rho}{\rho_p}\right] v_t = \left(\frac{\rho_p - \rho}{\rho_p}\right) g - \frac{3\pi D_p \mu v}{m} = \frac{dv}{dt}.$$

for terminal velocity $\frac{dv}{dt} = 0$, $v = v_t$.

D_p = from sphericity

$$m = \rho_p \times V \quad \underset{\searrow}{} \; \frac{\pi D_p^3}{6}$$

$$\Rightarrow \left(\frac{\rho_p - \rho}{\rho_p}\right) g - \frac{3\pi D_p \mu v}{\rho_p \cdot \frac{\pi D_p^3}{6}} = 0$$

$$\Rightarrow \left(\frac{\rho_p - \rho}{\rho_p}\right) g - \frac{18 \mu v_t}{\rho_p D_p^2} = 0$$

$$\boxed{v_t = \frac{D_p^2 g (\rho_p - \rho)}{18\mu}}$$ For laminar flow

$$\boxed{v_t \propto D_p^2}$$

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Criteria of settling Regime :-

I- Case

$$N_{Re} = \frac{\rho V_t (D) \, y^L}{\mu}$$

$$V_t = \frac{g D_p^2 (\rho_p - \rho)}{18 \mu} \quad \text{(from laminar eq}^n \text{ Stoke's eq}^n\text{)}$$

$$\Rightarrow N_{Re} = \frac{\rho g D_p^2 (\rho_p - \rho) D}{18 \mu \cdot \mu}$$

$$= \frac{g D_p^3 (\rho_p - \rho) \rho}{18 \mu^2}$$

$$\Rightarrow Re = \frac{K^3}{18}$$

$$K^3 = 18 \, Re$$

$$K = \sqrt[3]{18 \, Re}$$

$$\boxed{K = 2.62 \, Re^{1/3}}$$

$$\Rightarrow K = 2.62.$$

where $\boxed{K = D_p \left[\frac{g \rho (\rho_p - \rho)}{\mu^2} \right]^{1/3}}$.

If K is 2.62 to 4 then Stoke's Eq^n is used (i.e. flow is laminar).

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for Newton's range :-

$N_{Re} = 1.74\, K^{1.5}$

Reynold's no → 10000 $K = 69.12$

if $K > 4$ then Newton's law applies.

Hindered Settling :- In hindered settling a large no. of particles are present and surrounding particles interfere with the motion of individual particles, the vel. gradient surrounding each particle are effected by the closed tnu of other particles. The particles in settling in the liquid displaces the liquid and appreciable upward velocity of the liq. is generated. Hence the velocity of liq. is appreciable greater w.r.t. particle. For hindered settling, the settling velocity is less than the velocity as calc. from the Stoke's law.

Mathematically

$$\boxed{V_t = \frac{g\, D_p^2 (\rho_p - \rho)\, \epsilon}{18 (\mu / \psi_p)}}$$

ϵ = vol. fraction of slurry mixture occupied by
the liquid.

φ_p = correction factor

ϵ & φ_p → dimensionless no.

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Hindered Settling (Mcabe Smith). (15)

Sol: D_p for 100 - mesh = .147 mm

" " 80 - mesh = .175 mm

Avg. D_p = .161 mm = $.161 \times 10^{-3}$ m

Now, $\mu = .801$ cP

$= .801 \times 10^{-3}$ N-s/m²

$\rho_p = 2800$ kg/m³

$\rho = 1000$, [995.7 kg/m³].

Now, acc. to settling criteria for laminar ~~flow~~ calc. 'K'

$$K = D_p \left[\frac{g \rho (\rho_p - \rho)}{\mu^2} \right]^{1/3}$$

$= 4.86$

$K < 2.6 \rightarrow$ Stoke's Law

$K > 68.9$ then Newton

Now if K is more than ~~equal~~ ~~to 2.62 then~~ if $2.6 < K < 68.9$

~~Stoke's law applies.~~ Stokes range

Now, Acc. to Newton's ~~Stokes~~ law

$$V_t = \sqrt{\frac{4g(\rho_p - \rho) D_p}{3 C_D \rho}}$$

Assume $Re = 5$ then $C_D \cong 14$.

$\Rightarrow V_t = .0165$ m/sec.

Check: $Re = \frac{\rho V d}{\mu} = 3.30$ is more than 14. So now assume

∴ C_D at $Re = 3.30$

$Re = 2.5$ then $C_D \simeq 20$

$\Rightarrow V_t = .0138$ m/sec

Check $Re = 2.76$

Now it is near to the assumed value.

$\therefore V_t = .014$ m/s.

(b) Now, $a_e = 50g$.

$\therefore K = 17.90$

This is still in intermediate settling range.

Now assume $Re = 40$, $C_D = 4.1$

$V_t = .216$ m/sec

Check: $Re = \dfrac{\rho V d}{\mu} = 43 \simeq 40$

$\therefore \boxed{V_t = .22 \text{ m/sec}}$

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$F_D\ C_D = \frac{1}{2} \rho u^2 C_D . A$ (Re should be high)

Drag force

Sphere

$$\boxed{C_D = \frac{24}{Re} + \frac{2.6\left(\frac{Re}{5.0}\right)}{1+\left(\frac{Re}{5}\right)^{1.52}} + \frac{.411\left(\frac{Re}{263000}\right)^{-7.94}}{1+\left(\frac{Re}{263000}\right)^{-8.00}} + \left[\frac{Re^{.80}}{461000}\right]}$$