SIR CHHOTU RAM INSTITUTE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

ENGINEERING MECHANICS (BT-419)

NOTES ON CENTROID

This point, through which the whole weight of the body acts is known as **centre of gravity** (briefly written as C.G.). It may be noted that every body has one and only one centre of gravity. The centre of gravity depends upon the shape of the body.

14.2. Centroid

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre **of area of plane figures** is known as **centroid**. The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body. In many books, the authors also write centre of gravity for centroid and vice versa.

14.3. Methods for Centre of Gravity

The centre of gravity (or centroid) may be found out by any one of the following two methods:

1. By geometrical considerations
2. By moments
3. By graphical method

As a matter of fact, the graphical method is a tedious and cumbersome method for finding out the centre of gravity of simple figures. That is why, it has academic value only. But in this book, we shall discuss the procedure for finding out the centre of gravity of simple figures by geometrical considerations and by moments one by ones.

14.4. Centre of Gravity by Geometrical Considerations

The centre of gravity of simple figures may be found out from the geometry of the figure as given below.

Plane Area Figures

1. The centre of gravity of a rectangle (or a parallelogram) is at the point, where its diagonals meet each other. It is also a middle point of the length as well as the breadth of the rectangle as shown in Fig. 14.1.

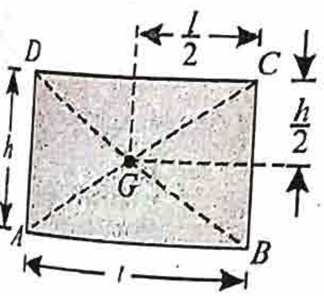

Fig. 14.1. Rectangle

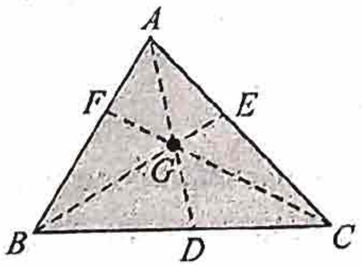

Fig. 14.2. Triangle

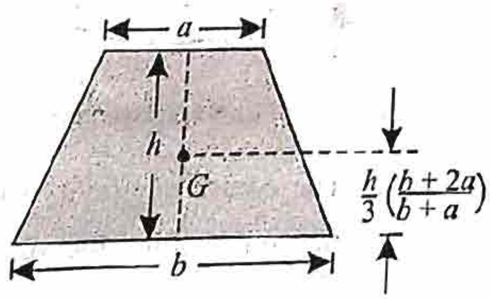

Fig. 14.3. Trapezium

2. The centre of gravity of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig. 14.2.
3. The centre of gravity of a trapezium with parallel sides a and b is at a distance of $\frac{h}{3} \times \left(\frac{b + 2a}{b + a}\right)$ measured form the side b as shown in Fig. 14.3.
4. The centre of gravity of a semicircle is at a distance of $\frac{4r}{3\pi}$ from its base measured along the vertical radius as shown in Fig. 14.4 (a).

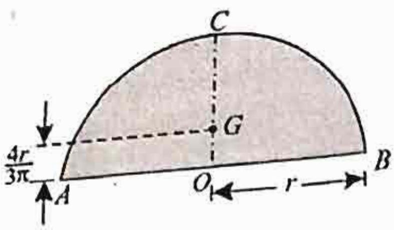

Fig. 14.4 (a). Semicircle

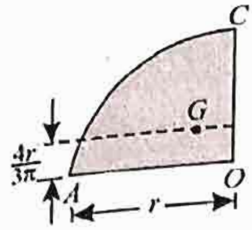

Fig. 14.4 (b). Quarter-circle

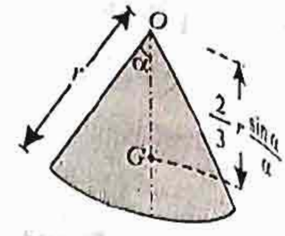

Fig. 14.4 (c). Circular sector

4. The centre of gravity of a quarter-circle is at a distance of $\frac{4r}{3\pi}$ from its base, measured along the vertical radius, as shown in Fig. 14.4 (b).
5. The centre of gravity of a circular sector making semi-vertical angle α is at a distance of $\frac{2r}{3}\frac{\sin\alpha}{\alpha}$ from the centre of the sector, measured along the central axis, as shown in Fig. 14.4 (c).

Solid Figures

6. The centre of gravity of a cube is at a distance of $\frac{l}{2}$ from every face (where l is the length of each side).
7. The centre of gravity of a cylinder is at a distance of $\frac{h}{2}$ from from its base (where h is the hieght of cylinder), as shown in Fig. 14.5.

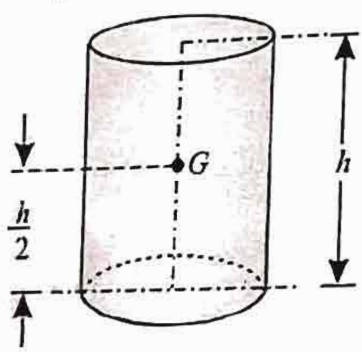

Fig. 14.5. Cylinder

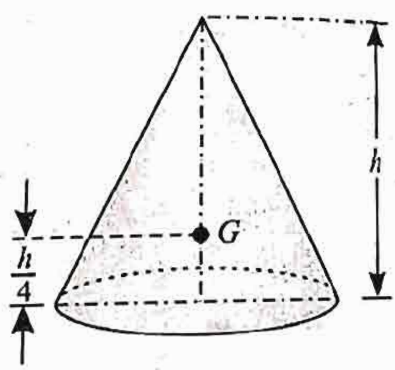

Fig. 14.6. Right circular solid cone

8. The centre of gravity of right circular solid cone is at a distance of $\frac{h}{4}$ from its base, measured along the vertical axis (where h is the hieght of cylinder), as shown in Fig. 14.6.
9. The centre of gravity of a sphere is at a distance of $\frac{d}{2}$ from every point (where d is the diameter of the sphere).

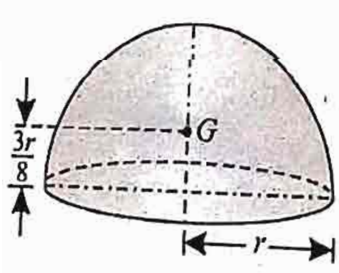

Fig. 14.7. Hemisphere

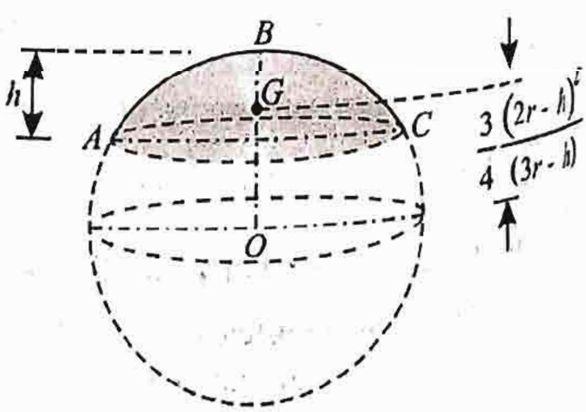

Fig.14.8. Segment of a sphere

10. The centre of gravity of a hemisphere is at a distance of $\frac{3r}{8}$ from its base, measured along the vertical radius, as shown in Fig. 14.7.

11. The centre of gravity of a segment of sphere of a height h is at a distance of $\frac{3}{4}\frac{(2r-h)^2}{(3r-h)}$ from the centre of the sphere, measured along the height, as shown in Fig. 14.8.

14.5. Centre of Gravity by Moments

The centre of gravity of a body may also be found out by moments as discussed below:

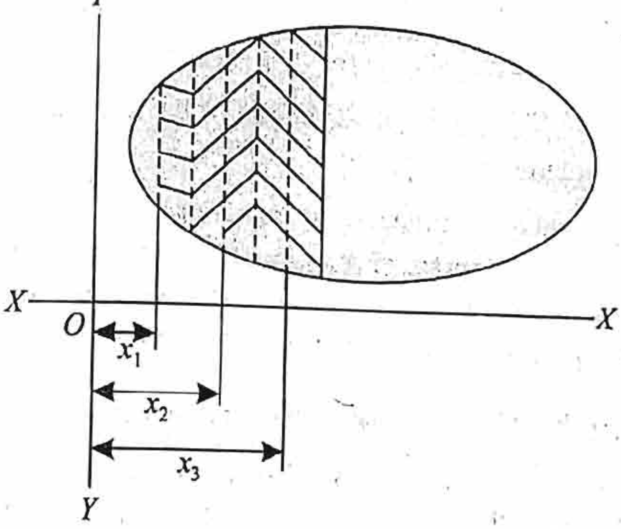

Fig. 14.9. Centre of gravity by moments

Consider a body of mass M whose centre of gravity is required to be found out. Divide the body into small masses, whose centres of gravity are known as shown in Fig. 14.9. Let m_1, m_2, m_3....; etc. be the masses of the particles and $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, ... be the co-ordinates of the centres of gravity from a fixed point O as shown in Fig. 14.9.

Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centre of gravity of the body. From the principle of moments, we know that

$$M\bar{x} = m_1 x_1 + m_2 x_2 + m_3 x_3 \ .....$$

or

$$\bar{x} = \frac{\Sigma mx}{M}$$

Similarly

$$\bar{y} = \frac{\Sigma my}{M}$$

where

$$M = m_1 + m_2 + m_3 + \ .....$$

14.6. Axis of Reference

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference (or sometimes with reference to some point of reference). The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating $\bar{y}$ and the left line of the figure for calculating $\bar{x}$.

14.7. Centre of Gravity of Plane Figures

The plane geometrical figures (such as *T*-section, *I*-section, *L*-section etc.) have only areas but no mass. The centre of gravity of such figures is found out in the same way as that of solid bodies. The centre of area of such figures is known as centroid, and coincides with the centre of gravity of the figure. It is a common practice to use centre of gravity for centroid and vice versa.

Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centre of gravity with respect to some axis of reference,

then
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots}{a_1 + a_2 + a_3}$$

and
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots\ldots}{a_1 + a_2 + a_3 + \ldots}$$

where a_1, a_2, a_3........ etc., are the areas into which the whole figure is divided x_1, x_2, x_3..... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on X-X axis with respect to same axis of reference.

y_1, y_2, y_3....... etc., are the respective co-ordinates of the areas a_1, a_2, a_3....... on Y-Y axis with respect to same axis of the reference.

Note. While using the above formula, x_1, x_2, x_3 ..... or y_1, y_2, y_3 or $\bar{x}$ and $\bar{y}$ must be measured from the same axis of reference (or point of reference) and on the same side of it. However, if the figure is on both sides of the axis of reference, then the distances in one direction are taken as positive and those in the opposite directions must be taken as negative.

14.8. Centre of Gravity of Symmetrical Sections

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about X-X axis or Y-Y axis. In such cases, the procedure for calculating the centre of gravity of the body is very much simplified; as we have only to calculate either $\bar{x}$ or $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

Example 14.1. *Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.*

Solution. As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles *ABCH* and *DEFG* as shown in Fig 14.10.

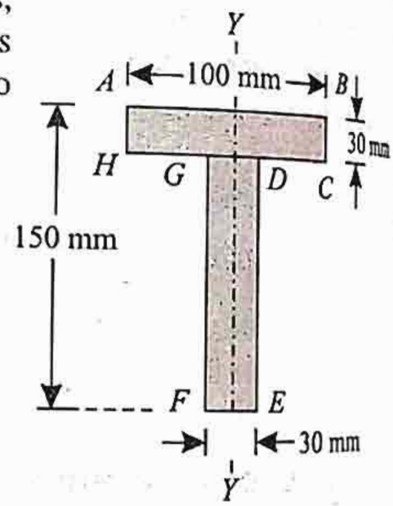

Fig.14.10.

Let bottom of the web *FE* be the axis of reference.

(*i*) *Rectangle ABCH*

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and,
$$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(*ii*) *Rectangle DEFG*

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

and
$$y_2 = \frac{120}{2} = 60 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange *FE*,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$= 94.1$ mm **Ans.**

Example 14.2. *Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.*

Solution. As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles *ABFJ*, *EGKJ* and *CDHK* as shown in Fig. 14.11.

Let the face *AC* be the axis of reference.

(i) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and
$$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(ii) *Rectangle EGKJ*

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and
$$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(iii) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and
$$x_3 = \frac{50}{2} = 25 \text{ mm}$$

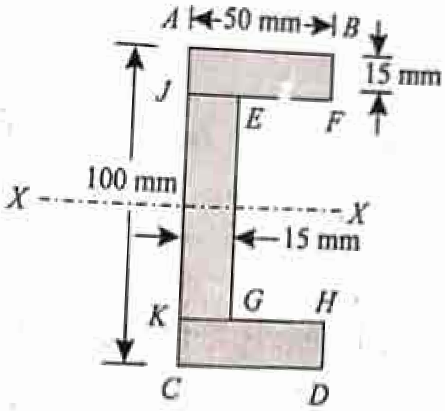

Fig.14.11.

We know that distance between the centre of gravity of the section and left face of the section *AC*,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

Example 14.3. *An I-section has the following dimensions in mm units :*

Bottom flange = 300 × 100

Top flange = 150 × 50

Web = 300 × 50

Determine mathematically the position of centre of gravity of the section.

Solution. As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in Fig. 14.12.

Let bottom of the bottom flange be the axis of reference.

(i) *Bottom flange*

$$a_1 = 300 \times 100 = 30\,000 \text{ mm}^2$$

and
$$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(ii) *Web*

$$a_2 = 300 \times 50 = 15\,000 \text{ mm}^2$$

and
$$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

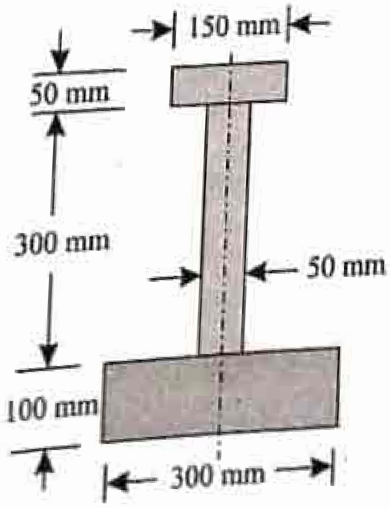

Fig.14.12.

(iii) *Top flange*

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_3 = 100 + 300 + \frac{50}{2} = 425 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30\,000 \times 50) + (15\,000 \times 250) + (7500 \times 425)}{30\,000 + 15\,000 + 7500} = 160.7 \text{ mm}$$ **Ans.**

14.9. Centre of Gravity of Unsymmetrical Sections

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to find out both the values of $\bar{x}$ and $\bar{y}$

Example 14.4. *Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.*

Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles as shown in Fig. 14.13.

Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(i) *Rectangle* 1

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(ii) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}.$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

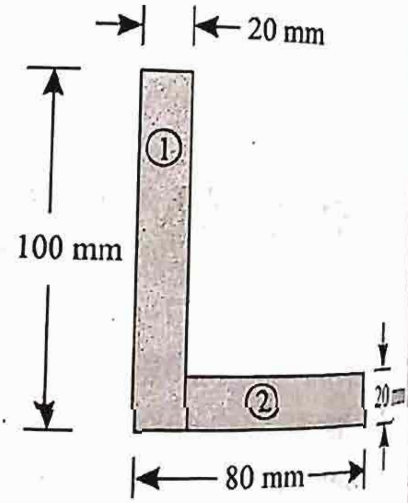

Fig.14.13.

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm}$$ **Ans.**

Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm}$$ **Ans.**

Example 14.5. *A uniform lamina shown in Fig. 14.14 consists of a rectangle, a circle and a triangle.*

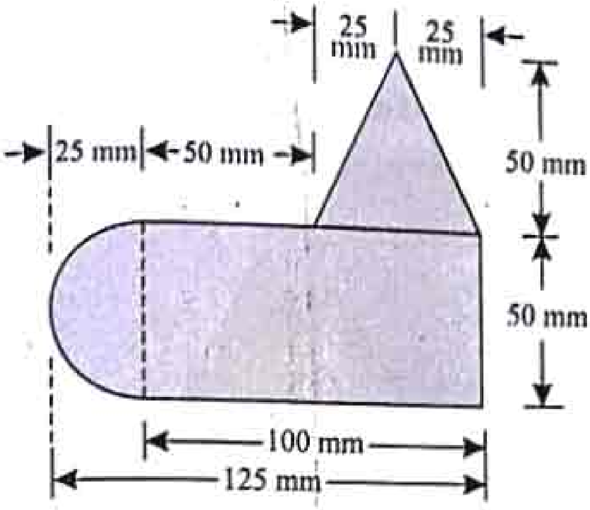

Fig. 14.14.

Determine the centre of gravity of the lamina. All dimensions are in mm.

Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of both $\bar{x}$ and $\bar{y}$ for the lamina.

Let left edge of circular portion and bottom face rectangular portion be the axes of reference.

(i) *Rectangular portion*

$$a_1 = 100 \times 50 = 5000 \text{ mm}^2$$

$$x_1 = 25 + \frac{100}{2} = 75 \text{ mm}$$

and $$y_1 = \frac{50}{2} = 25 \text{ mm}$$

(ii) *Semicircular portion*

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(25)^2 = 982 \text{ mm}^2$$

$$x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4 \text{ mm}$$

and $$y_2 = \frac{50}{2} = 25 \text{ mm}$$

(iii) *Triangular portion*

$$a_3 = \frac{50 \times 50}{2} = 1250 \text{ mm}^2$$

$$x_3 = 25 + 50 + 25 = 100 \text{ mm}$$

and $$y_3 = 50 + \frac{50}{3} = 66.7 \text{ mm}$$

We know that distance between centre of gravity of the section and left edge of the circular portion,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250}$$

$$= 71.1 \text{ mm} \qquad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face of the rectangular portion,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250}$$

$= 32.2$ mm **Ans.**

Example 14.6. *A plane lamina of 220 mm radius is shown in figure given below*

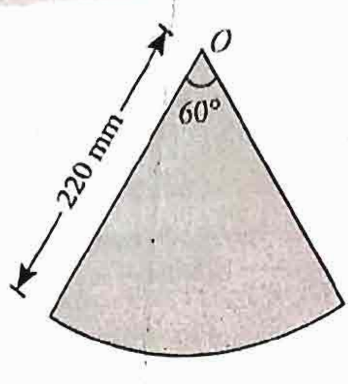

Fig. 14.15.

Find the centre of gravity of lamina from the point O.

Solution. As the lamina is symmetrical about *y-y* axis, bisecting the lamina, therefore its centre of gravity lies on this axis. Let *O* be the reference point. From the geometry of the lamina, we find that semi-vertical angle of the lamina

$$\alpha = 30° = \frac{\pi}{6} \text{ rad}$$

We know that distance between the reference point *O* and centre of gravity of the lamina,

$$\bar{y} = \frac{2r}{3}\frac{\sin\alpha}{\alpha} = \frac{2 \times 220}{3} \times \frac{\sin 30°}{\frac{\pi}{6}} = \frac{440}{3} \times \frac{0.5}{\frac{\pi}{6}} = 140 \text{ mm} \quad \textbf{Ans.}$$

EXERCISE 14.1

1. Find the centre of gravity of a *T*-section with flange 150 mm × 10 mm and web also 150 mm × 10 mm.
   [Ans. 115 mm for bottom of the web]
2. Find the centre of gravity of an inverted *T*-section with flange 60 mm × 10 mm and web 50 mm × 10 mm
   [Ans. 18.6 mm from bottom of the flange]
3. A channel section 300 mm × 10 mm is 20 mm thick. Find the centre of gravity of the section from the back of the web.
   [Ans. 27.4 mm]
4. Find the centre of gravity of an *T*-section with top flange 100 mm × 20 mm, web 200 mm × 30 mm and bottom flange 300 mm × 40 mm.
   [Ans. 79 mm from bottom of lower flange]
5. Find the position of the centre of gravity of an unequal angle section 10 cm × 16 cm × 2cm.
   [Ans. 5.67 cm and 2.67 cm]

6. A figure consists of a rectangle having one of its sides twice the other, with an equilateral triangle described on the larger side. Show that centre of gravity of the section lies on the line joining the rectangle and triangle.

7. A plane lamina of radius 100 mm as shown in fig 14.16 given below:

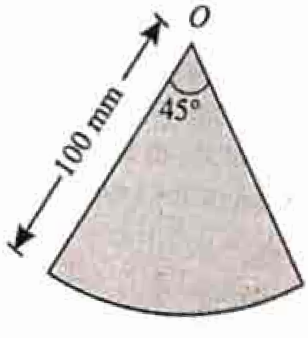

Fig. 14.16.

Find the centre of gravity of lamina from the point *O*.

[Ans. 65 mm]

14.10. Centre of Gravity of Solid Bodies

The centre of gravity of solid bodies (such as hemispheres, cylinders, right circular solid cones etc.) is found out in the same way as that of plane figures. The only difference, between the plane figures and solid bodies, is that in the case of solid bodies, we calculate volumes instead of areas. The volumes of few solid bodies are given below :

1. Volume of cylinder $= \pi \times r^2 \times h$
2. Volume of hemisphere $= \dfrac{2\pi}{3} \times r^3$
3. Volume of right circular solid cone $= \dfrac{\pi}{3} \times r^2 \times h$

where r = Radius of the body, and

h = Height of the body.

Note. Sometimes the densities of the two solids are different. In such a case, we calculate the weights instead of volumes and the centre of gravity of the body is found out as usual.

Example 14.7. *A solid body formed by joining the base of a right circular cone of height H to the equal base of a right circular cylinder of height h. Calculate the distance of the centre of mass of the solid from its plane face, when H = 120 mm and h = 30 mm.*

Solution. As the body is symmetrical about the vertical axis, therefore its centre of gravity lies on this axis as shown in Fig. 14.17. Let r be the radius of the cylinder base in cm. Now let base of cylinder be the axis of reference.

(i) *Cylinder*

$$v_1 = \pi \times r^2 \times 30 = 30\,\pi\, r^2\,\text{mm}^3$$

and

$$y_1 = \frac{30}{2} = 15 \text{ mm}$$

(ii) *Right circular cone*

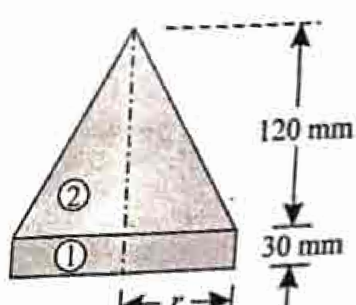

Fig.14.17.

$$v_2 = \frac{\pi}{3} \times r^2 \times h = \frac{\pi}{3} \times r^2 \times 120 \text{ mm}^3$$

$$= 40\,\pi r^2 \text{ mm}^3$$

and

$$y_2 = 30 + \frac{120}{4} = 60 \text{ mm}$$

We know that distance between centre of gravity of the section and base of the cylinder,

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{(30\pi r^2 \times 15) + (40\pi r^2 \times 60)}{30\pi r^2 + 40\pi r^2} = \frac{2850}{70} \text{ mm}$$

$$= 40.7 \text{ mm} \quad \textbf{Ans.}$$

Example 14.8. *A body consists of a right circular solid cone of height 40 mm and radius 30 mm placed on a solid hemisphere of radius 30 mm of the same material. Find the position of centre of gravity of the body.*

Solution. As the body is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis as shown in Fig. 14.18. Let bottom of the hemisphere (*D*) be the point of reference.

(*i*) *Hemisphere*

$$v_1 = \frac{2\pi}{3} \times r^3 = \frac{2\pi}{3}(30)^3 \text{ mm}^3$$

$$= 18\,000\,\pi \text{ mm}^3$$

and

$$y_1 = \frac{5r}{8} = \frac{5 \times 30}{8} = 18.75 \text{ mm}$$

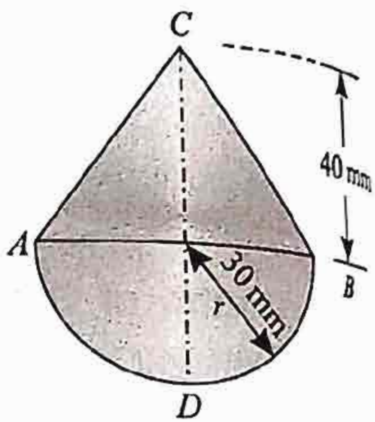

Fig.14.18.

(*ii*) *Right circular cone*

$$v_2 = \frac{\pi}{3} \times r^2 \times h = \frac{\pi}{3} \times (30)^2 \times 40 \text{ mm}^3$$

$$= 12\,000\,\pi \text{ mm}^3$$

and

$$y_2 = 30 + \frac{40}{4} = 40 \text{ mm}$$

We know that distance between centre of gravity of the body and bottom of hemisphere *D*,

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{(18\,000\pi \times 18.75) + (12\,000\pi \times 40)}{18\,000\pi + 12\,000\pi} \text{ mm}$$

$$= 27.3 \text{ mm} \quad \textbf{Ans.}$$

14.11. Centre of Gravity of Sections with Cut out Holes

The centre of gravity of such a section is found out by considering the main section, first as a complete one, and then deducting the area of the cut out hole *i.e.*, by taking the area of the cut out hole as negative. Now substituting a_2 (*i.e.*, the area of the cut out hole) as negative, in the general equation for the centre of gravity, we get

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} \quad \text{and} \quad \bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

Note. In case of circle the section will be symmeterical along the line joining the centres of the bigger and the cut out circle.

Example 14.12. *A square hole is punched out of circular lamina, the digonal of the square being the radius of the circle as shown in Fig.14.22. Find the centre of gravity of the remainder, if r is the radius of the circle.*

Solution. As the section is symmetrical about *X-X* axis, therefore its centre of gravity will lie on this axis. Let *A* be the point of reference.

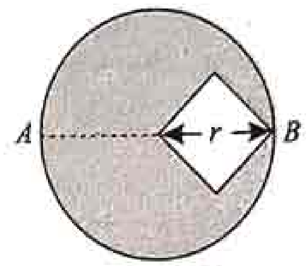

Fig.14.22.

(*i*) *Main circle*

$$a_1 = \pi r^2$$

and

$$x_1 = r$$

(*ii*) *Cut out square*

$$a_2 = \frac{r \times r}{2} = 0.5\, r^2$$

and

$$x_2 = r + \frac{r}{2} = 1.5\, r$$

We know that distance between centre of gravity of the section and *A*,

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(\pi r^2 \times r) - (0.5 r^2 \times 1.5 r)}{\pi r^2 - 0.5 r^2}$$

$$= \frac{r^3(\pi - 0.75)}{r^2(\pi - 0.5)} = \frac{r(\pi - 0.75)}{\pi - 0.5} \quad \textbf{Ans.}$$

Example 14.13. *A semicircle of 90 mm radius is cut out from a trapezium as shown in Fig 14.23*

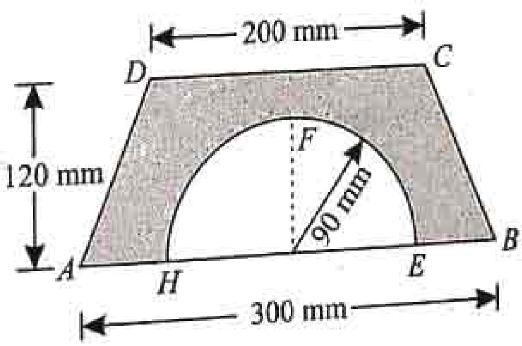

Fig. 14.23.

Find the position of the centre of gravity of the figure.

Solution. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Now consider two portions of the figure *viz.*, trapezium *ABCD* and semicircle *EFH*.

Let base of the trapezium *AB* be the axis of reference.

(*i*) *Trapezium ABCD*

$$a_1 = 120 \times \frac{200 + 300}{2} = 30\,000 \text{ mm}^2$$

and $$y_1 = \frac{120}{3} \times \left(\frac{300 + 2 \times 200}{300 + 200}\right) = 56 \text{ mm}$$

(ii) *Semicircle*

$$a_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times (90)^2 = 4050\pi \text{ mm}^2$$

and $$y_2 = \frac{4r}{3\pi} = \frac{4 \times 90}{3\pi} = \frac{120}{\pi} \text{ mm}$$

We know that distance between centre of gravity of the section and *AB*,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(30\,000 \times 56) - 4050\pi \times \frac{120}{\pi}}{30\,000 - 4050\pi} \text{ mm}$$

$$= 69.1 \text{ mm} \quad \textbf{Ans.}$$

Example 14.14. *A semicircular area is removed from a trapezium as shown in Fig.14.24 (dimensions in mm)*

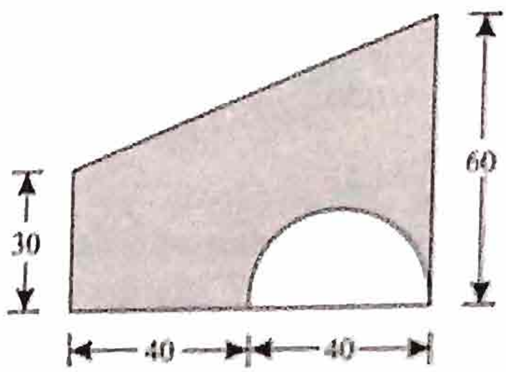

Fig. 14.24.

Determine the centroid of the remaining area (shown hatched). **(UPTU 2009-2010)**

Solution. As the section in not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the area. Split up the area into three parts as shown in Fig. 14.25. Let left face and base of the trapezium be the axes of reference.

(i) *Rectangle*

$$a_1 = 80 \times 30 = 2400 \text{ mm}^2$$

$$x_1 = \frac{80}{2} = 40 \text{ mm}$$

and $$y_1 = \frac{30}{2} = 15 \text{ mm}$$

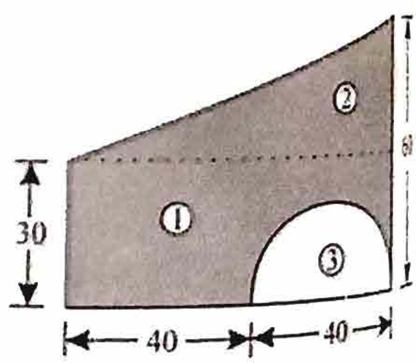

Fig.14.25.

(ii) *Triangle*

$$a_2 = \frac{80 \times 30}{2} = 1200 \text{ mm}^2$$

$$x_2 = \frac{80 \times 2}{3} = 53.3 \text{ mm}$$

and $$y_2 = 30 + \frac{30}{3} = 40 \text{ mm}$$

(iii) *Semicircle*

$$a_3 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(20)^2 = 628.3 \text{ mm}^2$$

$$x_3 = 40 + \frac{40}{2} = 60 \text{ mm}$$

and
$$y_3 = \frac{4r}{3\pi} = \frac{4 \times 20}{3\pi} = 8.5 \text{ mm}$$

We know that distance between centre of gravity of the area and left face of trapezium,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 40) + (1200 \times 53.3) - (628.3 \times 60)}{2400 + 1200 - 628.3}$$

$$= 41.1 \text{ mm} \quad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the area and base of the trapezium,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 15) + (1200 \times 40) - (628.3 \times 8.5)}{2400 + 1200 - 628.3}$$

$$= 26.5 \text{ mm} \quad \textbf{Ans.}$$

Example 14.15. *A circular sector of angle 60° is cut from the circle of radius r as shown in Fig. 14.26 :*

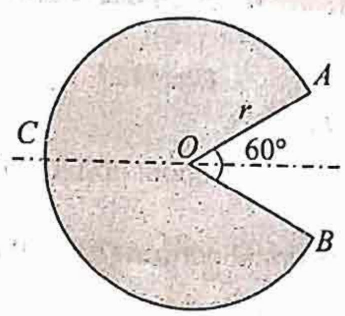

Fig. 14.26.

Determine the centre of gravity of the remainder.

Solution: As the section is symmetrical about *X-X* axis, therefore its centre of gravity will lie on this axis.

Let *C* be the reference point.

(*i*) *Main circle*

$$a_1 = \pi r^2$$

and
$$x_1 = r$$

(*ii*) *Cut out sector*

$$a_2 = \frac{\pi r^2 \theta}{360°} = \frac{\pi r^2 \times 60°}{360°} = \frac{\pi r^2}{6}$$

and
$$x_2 = r + \frac{2r}{\pi}$$

We know that distance between the centre of gravity of the section and *C*

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(\pi r^2 \times r) - \left[\frac{\pi r^2}{6} \times \left(r + \frac{2r}{\pi}\right)\right]}{\pi r^2 - \frac{\pi r^2}{6}}$$

$$= \frac{\pi r^2 \left[r - \frac{1}{6}\left(r + \frac{2r}{\pi}\right)\right]}{\pi r^2 \left(1 - \frac{1}{6}\right)} = \frac{r - \left[\frac{1}{6} \times \left(r + \frac{2r}{\pi}\right)\right]}{1 - \frac{1}{6}}$$

$$= \frac{6}{5}\left[r - \left(\frac{r}{6} + \frac{2r}{6\pi}\right)\right] = \frac{6}{5}\left[r - \frac{r}{6} - \frac{r}{3\pi}\right]$$

$$= \frac{6}{5}\left(\frac{5}{6}r - \frac{r}{3\pi}\right) = r - \frac{2r}{5\pi} \qquad \textbf{Ans.}$$

Example 14.16. *A solid consists of a right circular cylinder and a hemisphere with a cone cut out from the cylinder as shown in Fig. 14.27.*

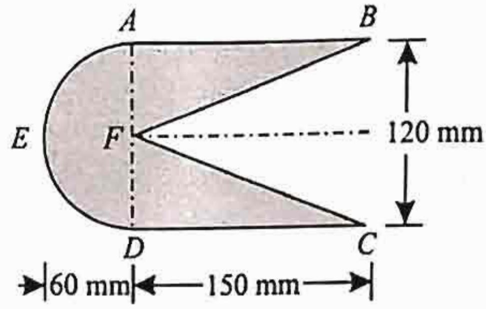

Fig. 14.27

Find the centre of gravity of the body.

Solution. As the solid is symmetrical about horizontal axis, therefore its centre of gravity lie on this axis.

Let the left edge of the hemispherical portion (E) be the axis of reference.

(*i*) *Hemisphere ADE*

$$v_1 = \frac{2\pi}{3} \times r^3 = \frac{2\pi}{3} \times (60)^3 = 144\,000\,\pi \text{ mm}^3$$

and $$x_1 = \frac{5r}{8} = \frac{5 \times 60}{8} = 37.5 \text{ mm}$$

(*ii*) *Right circular cylinder ABCD*

$$v_2 = \pi \times r^2 \times h = \pi \times (60)^2 \times 150 = 540\,000\,\pi \text{ mm}^3$$

and $$x_2 = 60 + \frac{150}{2} = 135 \text{ mm}$$

(*iii*) *Cone BCF*

$$v_3 = \frac{\pi}{3} \times r^2 \times h = \frac{\pi}{3} \times (60)^2 \times 150 = 180\,000\,\pi \text{ mm}^3$$

and $$x_3 = 60 + 150 \times \frac{3}{4} = 172.5 \text{ mm}$$

We know that distance between centre of gravity of the solid and left edge of the hemisphere (E),

$$\bar{x} = \frac{v_1 x_1 + v_2 x_2 - v_3 x_3}{v_1 + v_2 - v_3}$$

$$= \frac{(144\,000\,\pi \times 37.5) + (540\,000\,\pi \times 135) - (180\,000\,\pi \times 172.5)}{144\,000\,\pi + 540\,000\,\pi - 180\,000\,\pi}$$

$$= 93.75 \text{ mm} \qquad \textbf{Ans.}$$