

Class - B.Tech 2<sup>nd</sup> yr ECE  
Subject: Signal & System  
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## Topic: Properties of DFT

### 1. Linearity

Statement: If  $x_1(n) \xrightarrow[N]{\text{DFT}} X_1(k)$  and

$x_2(n) \xrightarrow[N]{\text{DFT}} X_2(k)$  then

$$a_1 x_1(n) + a_2 x_2(n) \xrightarrow[N]{\text{DFT}} a_1 X_1(k) + a_2 X_2(k)$$

here  $a_1$  and  $a_2$  are constants

Period Subject

Proof

Acc. to definition of DFT,

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{kn}$$

here  $x(n) = a_1 x_1(n) + a_2 x_2(n)$

$$\therefore X(k) = \sum_{n=0}^{N-1} [a_1 x_1(n) + a_2 x_2(n)] W_N^{kn}$$

$$= \sum_{n=0}^{N-1} a_1 x_1(n) W_N^{kn} + \sum_{n=0}^{N-1} a_2 x_2(n) W_N^{kn}$$

Since  $a_1$  and  $a_2$  are constants, we can take it out of the summation sign

$$\therefore X(k) = a_1 \sum_{n=0}^{N-1} x_1(n) W_N^{kn} + a_2 \sum_{n=0}^{N-1} x_2(n) W_N^{kn}$$

Comparing eq. (1) with definition of DFT.

$$X(k) = a_1 X_1(k) + a_2 X_2(k)$$

2. Periodicity

Statement:

If  $x(n) \xleftrightarrow[N]{\text{DFT}} X(k)$  then

$$x(n+N) = x(n) \quad \text{for all } n.$$

$$X(k+N) = X(k)$$

3. Duality Property

Statement: If  $x(n) \xleftrightarrow[N]{\text{DFT}} X(k)$ .

Then  $x(n) \xleftrightarrow[N]{\text{DFT}} N X\left[\frac{(-k)N}{N}\right]$

u. Circular Convolution:

$$\text{If } x_1(n) \xleftrightarrow[N]{\text{DFT}} X_1(k)$$

$$x_2(n) \xleftrightarrow[N]{\text{DFT}} X_2(k) \text{ then}$$

$$x_1(n) \textcircled{N} x_2(n) \xleftrightarrow[N]{\text{DFT}} X_1(k) \cdot X_2(k)$$

here  $\textcircled{N}$  indicates circular convolution

5. Time Reversal Property

$$\text{If } x(n) \xleftrightarrow[N]{\text{DFT}} X(k)$$

$$\begin{aligned} \text{then } x((-n))_N &= x(N-n) \xleftrightarrow[N]{\text{DFT}} X((-k)) \\ &= X(N-k) \end{aligned}$$

6. circular time shift

Statement: If  $x(n) \xleftrightarrow{DFT} X(k)$  then

$$x((n-l))_N \xleftrightarrow{DFT} X(k) e^{-j2\pi k l / N}$$

or  $x((n-l))_N \xleftrightarrow{DFT} X(k) W_N^{kl}$

7. circular frequency shift:

Statement: If  $x(n) \xleftrightarrow{DFT} X(k)$  then

$$x(n) e^{j2\pi l n / N} \xleftrightarrow{DFT} X((k-l))_N = X(k+l)$$

or  $x(n) e^{-j2\pi l n / N} \xleftrightarrow{DFT} X((k+l))_N = X(k-l)$

Remarks :

8 Circular Correlations!

Statement: If  $x(n) \xleftrightarrow[N]{DFT} X(k)$  and

$y(n) \xleftrightarrow[N]{DFT} Y(k)$  then

$$\widetilde{R_{xy}}(l) \xleftrightarrow[N]{DFT} R_{xy}(k) = X(k)Y^*(k)$$

9 Parseval Theorem!

Statement: If  $x(n) \xleftrightarrow[N]{DFT} X(k)$  and

$y(n) \xleftrightarrow[N]{DFT} Y(k)$

then

$$\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)Y^*(k)$$

Remarks :

Days  
Period

Subject

Home Work

## Complex Conjugate Property

Statement

$$\text{If } x(n) \xrightarrow[N]{\text{DFT}} X(k)$$

then

$$x^*(n) \xrightarrow[N]{\text{DFT}} X^*((-k)_N) = X^*(N-k)$$

## 11. Multiplication of Two sequences

If

$$x_1(n) \xrightarrow[N]{\text{DFT}} X_1(k) \text{ and}$$

$$x_2(n) \longleftrightarrow X_2(k) \text{ then}$$

$$x_1(n) \cdot x_2(n) \xrightarrow[N]{\text{DFT}} \frac{1}{N} [X_1(k) \otimes X_2(k)]$$