UNIT -2

LINEAR SHIFT INVARIENT SYSTEMS

PART-1

Classification of S/m:-

* Continuous time S/m
* discrete time S/m

(i) Cont. time S/m

$x(t) \longrightarrow \boxed{\text{Cont. Time S/m}} \longrightarrow y(t)$

Ex → BJT, MOSFET, OPAMP

$x(t)$ → Cont. time Signal

$y(t)$ → Cont ———

(ii) Discrete Time S/m

$x[n] \longrightarrow \boxed{\text{Discrete Time S/m}} \longrightarrow y[n]$

ex- Flipflop, RAM, microprocessor

$x[n]$ = Discrete time Signal

$y[n]$ = ———

Properties of S/m :-

(i) Linearity :- (Linear S/m or Non-Linear S/m)

Linear S/m :-

For cont. Time S/m

$x(t) \longrightarrow \boxed{\text{Cont. Time S/m}} \longrightarrow y(t)$

(i) case I

$x_1(t) \longrightarrow \boxed{\text{S/m}} \longrightarrow y_1(t)$

Case II

$x_2(t) \longrightarrow \boxed{\text{S/m}} \longrightarrow y_2(t)$

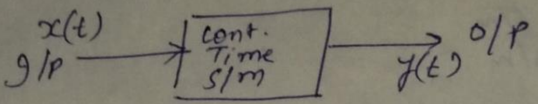
Case III

$[x_1(t) + x_2(t)] \longrightarrow \boxed{\text{S/m}} \longrightarrow \cancel{[y_1(t) + y_2(t)]}\ y(t)$

If $y(t) = y_1(t) + y_2(t)$ [Linear S/m]

$y(t) \neq y_1(t) + y_2(t)$ [non-linear S/m]

Ques:-



$$\boxed{y(t) = 5x(t)}$$

Solⁿ:-

Case I

$i/p = x_1(t)$

$o/p = y_1(t)$ Let

$y_1(t) = 5x_1(t)$ ——— ①

Case II :-

$i/p = x_2(t)$

$o/p = y_2(t)$ Let

$y_2(t) = 5x_2(t)$ ——— ②

Case III

$i/p = x_1(t) + x_2(t)$

$o/p = y(t) = ?$

$y(t) = 5x(t)$

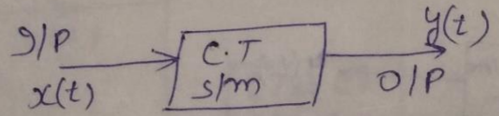
$= 5[x_1(t) + x_2(t)]$

$= 5x_1(t) + 5x_2(t)$

$y(t) = y_1(t) + y_2(t)$

So s/m is linear s/m

Ques:-



$$\boxed{y(t) = 9x^2(t)}$$

Nonlinear s/m

Ques $y(t) = 6x(t)+5$

Non linear

~~for~~ **Discrete Time s/m:-**

~~$y[n] = 6x[n]$~~

i/p $x[n]$ → [Discrete time s/m] → o/p $y[n]$

Case I
  I/p = $x_1[n]$
  o/p = $y_1[n]$

Case II
  i/p = $x_2[n]$
  o/p = $y_2[n]$

Case III
  i/p = $x_1[n] + x_2[n]$
  o/p = $y[n]$

if $y[n] = y_1[n] + y_2[n]$ → Linear

$y[n] \neq$ ——— → Non Linear

Ques $y[n] = 6x[n]$
Linear s/m

Ques $5x^2[n] + 7$
Non Linear

Property II :- Time Invariance (Time Invarient or Time Varient s/m)

~~Time Inva~~

Time Invarient S/m :- If there is no time lag b/w i/p & o/p then s/m is time Invarient s/m

For Cont. Time s/m

$x(t)$ → [Cont. Time s/m] → $y(t)$

$x(t-t_0)$ → [C.T. s/m] → $y(t-t_0)$

Procedure (1) First of all put $t \to (t-t_0)$ in Input $x(t)$ and o/p $y(t)$

(2) Put $t \to (t-t_0)$ in compleat relation of i/p & o/p

(3) If both remain same → Time Invarient
  are diffrent → Time Varient s/m

Ques

$$\underset{x(t)}{\text{i/p}} \longrightarrow \boxed{\begin{matrix}\text{C.T}\\ \text{s/m}\end{matrix}} \longrightarrow \underset{y(t)}{\text{o/p}}$$

$$y(t) = 3t^2 . x(t)$$

Determine

(1) wheather S/m is time Invarient or not

(2) ———————— linear or not

(3) ———————— LTI or not

Solⁿ:- ① (i)

$$y(t-t_0) = 3t^2 \{ x(t-t_0)$$

(ii) $$y(t-t_0) = 3(t-t_0)^2 \, x(t-t_0)$$

both are diffrent so → Time varient

②

$$y_1(t) = 3t^2 x_1(t) \quad \text{——— ①}$$

$$y_2(t) = 3t^2 \{ \cancel{y_2(t)} \; x_2(t) \quad \text{——— ②}$$

$$y t = 3t^2 (x_1(t) + x_2(t)]$$

$$= 3t^2 x_1(t) + 3t^2 x_2(t)$$

$$y(t) = y_1(t) + y_2(t)$$

So Linear s/m

③ S/m is linear but time varient therefor s/m is not LTI s/m.

Ques 2:- $y(t) = 4[x(t)]^2$ → Time Invarient non Linear

find for LTI.

Ques 3:- $y[n] = 6x[n]$ → Linear and Time Invarient so s/m is LTI.

Property 3:- Causality (causal and Non causal s/m)

Causal s/m:- Those s/m whose o/p depends upon present and/or past i/p of s/m.

Ex for Cont. time s/m

(i) $y(t) = 3x(t)$
(ii) $y(t) = 5x(t-2)$
(iii) $y(t) = 6x(t) + 2x(t-3)$
(iv) $y(t) = \int_0^t x(\tau)\, d\tau$

For discrete Time s/m

(i) $y[n] = 4x[n]$
(ii) $y[n] = 2x[n-3]$
(iii) $y[n] = 5x[n-2] + 6x[n]$
(iv) $y[n] = \sum_{k=0}^{n} x[k]$

Non Causal s/m:- Those s/m whose o/p depends upon future i/p of s/m, s/m is non causal s/m.

Ex (i) $y(t) = 5x(t+2)$
(ii) $y(t) = 5x(t) + 3x(t+1)$
(iii) $y(t) = 3x(t) + 2x(t+1)$

Property 4:- Memory (Memory and memoryless s/m)

Memory s/m:- Those s/m whose o/p depends upon previous value of i/p is called memory s/m.

[Note:- Memory s/m is subset of causal s/m]

Ex for C.T s/m

(i) $y(t) = 4x(t-2)$
(ii) $y(t) = 2x(t) + 5x(t-3)$
(iii) $y(t) = \int_{-\infty}^{t} x(t)\, dt$

for D.T s/m

(i) $y[n] = 5x[n-3]$
(ii) $y[n] = 2x[n] + 3x[n-2]$
(iii) $y[n] = \sum_{k=-\infty}^{n} x[k]$

Memoryless S/m :- Those S/m whose o/p depends upon present value of i/p only.

Ex : $y(t) = 5x(t)$
$y[n] = 3x[n]$

Property 5: Stablity (Stable and Unstable S/m)

Stable S/m :- A S/m is said to be BIBO (Bounded input bounded o/p) stable if

(i) if i/p of S/m is finite then o/p of s/m is finite

(ii) If i/p of s/m is zero then o/p must be zero.

Ex (1) $y(t) = 5x(t)$

follows Ist and IInd Condition So S/m is Stable.

(II) $y(t) = 3x(t)+5$

follows Ist Cond. but do not follow IInd condition So unstable.

(III) $y(t) = 4 \cdot e^{x(t)}$

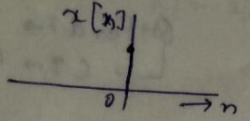
follows Ist but fails IInd So S/m is Unstable

Analysis of LTI S/m

(1) Representation of Discrete time signal

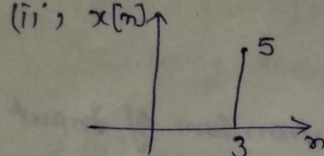
Basic :- A discrete time signal can be represented by Sum of Impulse signals.

Ex (i)



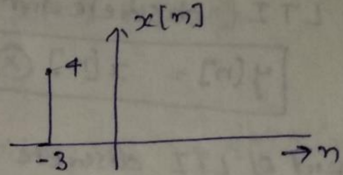
$x[n] = \delta[n]$

(ii)



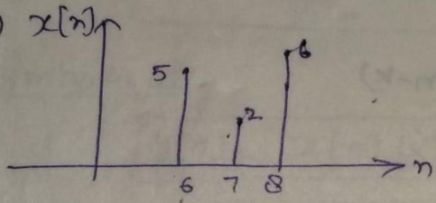
$x[n] = 5\,\delta[n-3]$

(iii)



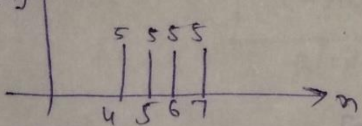
$x[n] = 4\,\delta[n+3]$

(iv)



$x[n] = 5\,\delta[n-6] + 2\,\delta[n-7] + 6\,\delta[n-8]$

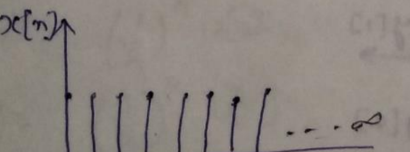
(v)



$x[n] = 5\,\delta[n-4] + 5\,\delta[n-5] + 5\,\delta[n-6] + 5\,\delta[n-7]$

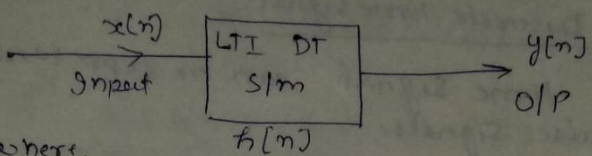
$x[n] = 5 \sum_{k=4}^{7} \delta(n-k)$

(vi)



$x[n] = \sum_{k=0}^{\infty} \delta(n-k) \quad = u[n]$

LTI Discrete Time S/m :-



where,

$h[n]$ = s/m transfer fuⁿ

For d.T → Conv. Sum
→ CT → Conv. Integral

For LTI discrete time s/m

$$\boxed{y[n] = x[n] \otimes h[n]}$$

" Output of LTI discrete time s/m is Convolution Sum of Input and transfer function."

Mathematically,

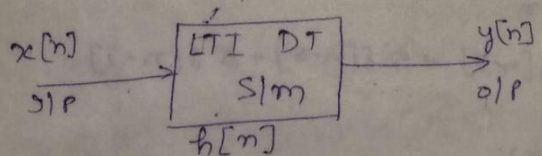
$$y[n] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

where,

$$x[k] = x[n]\Big|_{n \to k}$$

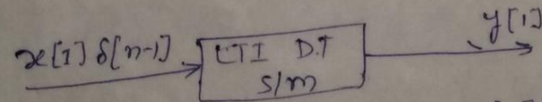
$$\& \quad h[n-k] = h[n]\Big|_{n \to (n-k)}$$

Proof

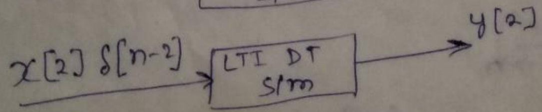


if s/m is Time Invarient [TI]

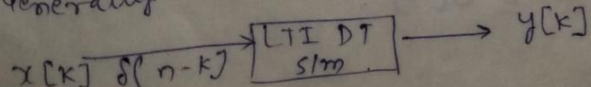
(i)



(ii)



(iii) Generally



If s/m is linear s/m

$$y[n] = \cdots\cdots y[1] + y[2] + \cdots + y[k] + \cdots\cdots \quad \text{——} (1)$$

where,

$$y[1] = x[1]\,\delta[n-1] \times h[n] = x[1]\,h[n-1]$$

$$y[2] = x[2]\,\delta[n-2] \times h[n] = x[2]\,h[n-2]$$

$$\vdots$$

In General

$$y[k] = x[k]\,\delta[n-k] \times h[n] = x[k]\,h[n-k]$$

Putting these values in eqn (1)

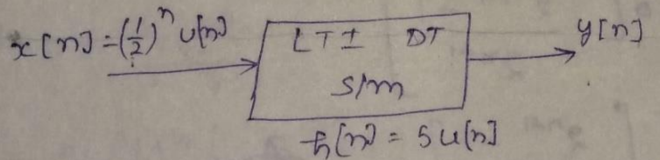
$$y[n] = \cdots + x[1]\,h[n-1] + x[2]\,h[n-2] + \cdots + x[k]\,h[n-k] + \cdots$$

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]}$$

Symbolically

$$\boxed{y[n] = x[n] \otimes h[n]}$$

Ques :-



Solⁿ :-

$$y[n] = x[n] \otimes h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$u(n-k) \to 1$ (+ve), $n-k \geq 0$, $\boxed{n \geq k}$

$u(n-k) \to 0$ (-ve), $n-k < 0$, $\boxed{k > n}$

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$x[k] = \left(\frac{1}{2}\right)^k u[k] = \begin{cases} 0, & k < 0 \\ \left(\frac{1}{2}\right)^k & k \geq 0 \end{cases}$$

$$h[n] = 5\,u[n]$$

$$h[n-k] = 5\,u[n-k] = \begin{cases} 0, & k > n \\ 5, & k \leq n \end{cases}$$

$$y[n] = \sum_{-\infty}^{-1} \underbrace{x[k]}_{0}\, h[n-k] + \sum_{k=0}^{n} x[k]\, h[n-k] + \sum_{k=n+1}^{\infty} \underbrace{x[n]\, h[n-k]}_{0}$$

$$y[n] = \sum_{k=0}^{n} x[k] \cdot h[n-k]$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \cdot 5$$

$$= 5\left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^n\right]$$

$$S_n = \frac{a[1-r^n]}{1-r} \quad \text{G.P} \quad [n \rightarrow \text{total no of terms}]$$

$$y[n] = 5\left[\frac{1\left(1-\left(\frac{1}{2}\right)^{n+1}\right)}{1-\frac{1}{2}}\right] = 5\left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1/2}\right]$$

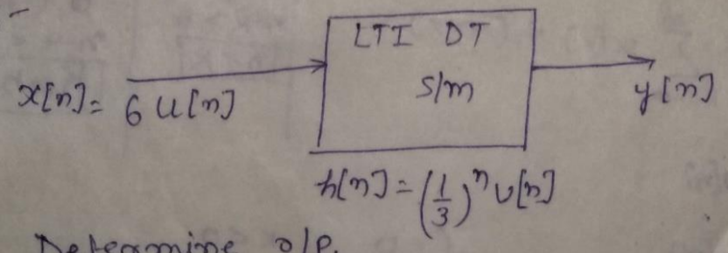
$$y[n] = 10\left[1-\left(\frac{1}{2}\right)^{n+1}\right]$$

$$= 10\left[1 - \frac{1}{2^{n+1}}\right]$$

$$= 10\left[\frac{2^{n+1}-1}{2^{n+1}}\right]$$

$$y[n] = 5\left[\frac{2^{n+1}-1}{2^n}\right]$$ Ans

Ques :-



Determine o/p.

Soln :- $$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$x[n] = 6.u[n]$

$x[k] = 6.u[k] = \begin{cases} 0 & k<0 \\ 6 & k \geq 0 \end{cases}$

$h[n] = \left(\frac{1}{3}\right)^n u[n]$

$h[n-k] = \left(\frac{1}{3}\right)^{n-k} u[n-k] = \begin{cases} 0 & k>n \\ \left(\frac{1}{3}\right)^{n-k} & k \leq n \end{cases}$

$$y[n] = \sum_{k=-\infty}^{-1} \underbrace{x[k]}_{0} h[n-k] + \sum_{k=0}^{n} x[k] . h[n-k] + \sum_{k=n+1}^{\infty} x[k] \underbrace{h[n-k]}_{0}$$

$$y[n] = \sum_{k=0}^{n} x[k] h[n-k]$$

$$= \sum_{k=0}^{n} 6 \left(\frac{1}{3}\right)^{n-k}$$

$$y[n] = 6\left[\left(\frac{1}{3}\right)^n + \left(\frac{1}{3}\right)^{n-1} + \left(\frac{1}{3}\right)^{n-2} + \dots + 1\right]$$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$= \left(\frac{1}{3}\right)^n \left[\frac{3^{n+1}-1}{3-1}\right]$$

$a = \left(\frac{1}{3}\right)^n$, $r = \frac{\left(\frac{1}{3}\right)^{n-1}}{\left(\frac{1}{3}\right)^n} = \frac{1}{\frac{1}{3}} = 3$

$$y[n] = 6 \cdot \left(\frac{1}{3}\right)^n \cdot \frac{3^{n+1}-1}{2}$$

$$= 3 \cdot \left(\frac{1}{3}\right)^n \left(3^{n+1}-1\right)$$

$$y[n] = \left(\frac{1}{3}\right)^{n-1} \cdot \left(3^{n+1}-1\right)$$ Ans

Ques)

$x[n] = \left(\frac{1}{3}\right)^n u[n] \longrightarrow$ LTI DT sys $\longrightarrow y[n]$

$h[n] = \left(\frac{1}{4}\right)^n u[n]$

Sol^n:-

$$y[n] = x[n] \circledast h[n]$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n]$$

$$x[k] = \left(\frac{1}{3}\right)^k u[k] = \begin{cases} 0 & k<0 \\ \left(\frac{1}{3}\right)^k & k \geq 0 \end{cases}$$

$$h[n] = \left(\frac{1}{4}\right)^n u[n]$$

$$h[n-k] = \left(\frac{1}{4}\right)^{n-k} u[n-k] = \begin{cases} 0 & k>n \\ \left(\frac{1}{4}\right)^{n-k} & k \leq n \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{-1} \underline{x[n]}\, h[n-k] + \sum_{k=0}^{n} x[k]\, h[n-k] + \sum_{k=n+1}^{\infty} x[k]\, \underline{h[n-k]}$$

$$y[n] = \sum_{k=0}^{n} x[n]\, h[n-k]$$

$$= \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \left(\frac{1}{4}\right)^{n-k} = \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \left(\frac{1}{4}\right)^{-k}$$

$$= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \frac{\left(\frac{1}{3}\right)^k}{\left(\frac{1}{4}\right)^k} = \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{4}{3}\right)^k$$

$$= \left(\frac{1}{4}\right)^n \left[1 + \frac{4}{3} + \left(\frac{4}{3}\right)^2 + \dots + \left(\frac{4}{3}\right)^n\right]$$

$$= \left(\frac{1}{4}\right)^n \cdot 1 \frac{\left[\left(\frac{4}{3}\right)^{n+1} - 1\right]}{\left(\frac{4}{3} - 1\right)} = \left(\frac{1}{4}\right)^n \times 3 \left\{\left(\frac{4}{3}\right)^{n+1} - 1\right\}$$

$$=$$

$$y[n] = 3\left(\frac{1}{4}\right)^n \left(\frac{4^{n+1} - 3^{n+1}}{3^{n+1}}\right) (12)$$

Ques

$$x[n] \longrightarrow \boxed{\text{LTI DT s/m}} \longrightarrow y[n]$$

$x[n] = 5u[n-6]$

$h[n] = \left(\frac{1}{3}\right)^n u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$x[n] = 5u[n-6]$

$$x[k] = 5u[k-6] = \begin{cases} 0 & k < 6 \\ 5 & k \geq 6 \end{cases}$$

$h[n] = \left(\frac{1}{3}\right)^n u[n]$

$$h[n-k] = \left(\frac{1}{3}\right)^{n-k} u[n-k] = \begin{cases} 0 & k > n \\ \left(\frac{1}{3}\right)^{n-k} & k \leq n \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{5} \underbrace{x[k]}_{0}\, h[n-k] + \sum_{k=6}^{n} x[k]\, h[n-k] + \sum_{k=n+1}^{\infty} x[k]\, \underbrace{h(n-k)}_{0}$$

$$y[n] = \sum_{k=6}^{n} x[k]\, h[n-k]$$

$$\cancel{y[n] = \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \left(\frac{1}{4}\right)^{n-k}}$$

$$\cancel{y[n] = \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k}$$

$$y[n] = \sum_{k=6}^{n} 5 \cdot \left(\frac{1}{3}\right)^{n-k}$$

$$= 5 \cdot \sum_{k=6}^{n} \left(\frac{1}{3}\right)^{n-k}$$

$$= 5 \cdot \left(\frac{1}{3}\right)^n \sum_{k=6}^{n} 3^k$$

$$= 5 \cdot \left(\frac{1}{3}\right)^n \left[3^6 + 3^7 + \cdots + 3^n\right]$$

$$= 5 \cdot \left(\frac{1}{3}\right)^n \cdot \frac{3^6\left[3^{n-5} - 1\right]}{3-1} = \frac{5}{2} \cdot \left(\frac{1}{3}\right)^{n-6} \cdot \left(3^{n-5} - 1\right)$$

Ans