Testing of Significance for Single Mean:-

To test whether the difference between sample mean and Population mean is significant or not under the null hypothesis that there is no difference between the sample mean and Population mean.

The test statistic is $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$, where σ is the standard deviation of the Population.

If σ is not known, we use the test statistic $z = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$, where s is the standard deviation of the sample.

Note:→ If the level of significance is α and z_α is the critical value

$$-z_\alpha < |z| = \left|\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right| < z_\alpha .$$

The limits of the Population mean μ are given by

$$\bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} .$$

At 5% level of significance, 95% confidence limits are

$$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}} .$$

At 1% level of significance, 99% confidence limits are

$$\bar{x} - 2.58 \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.58 \frac{\sigma}{\sqrt{n}} .$$

These limits are called confidence limits or fiducial limits.

Question!→ A random sample of 900 members has a mean 3.4 cms. Can it be reasonably regarded as a sample from a large Population of mean 3.2 cms and S.D. 2.3 cms?

Sol:- Here $n = 900$, $\bar{x} = 3.4$, $\mu = 3.2$

Null hypothesis:

H_0: Assume that the sample is drawn from a large Population with mean 3.2 and S.D. 2.3

Alternative hypothesis: $H_1 : \mu \neq 3.2$ (two tailed test)

Under H_0: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{3.4-3.2}{2.3/\sqrt{900}} = 0.261$

Conclusion:- As the calculated value of $|z| = 0.261 < 1.96$, the significant value of z at 5% level of significance.

Hence H_0 is accepted.

Imp.

Student's t - Distribution (t-Test):

This t-distribution is used when sample size is ≤ 30 and the Population standard deviation is unknown.

t - Statistic is defined as $t = \frac{\bar{x}-\mu}{S/\sqrt{n}}$, where $S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$

$\bar{x}$ is the mean of sample, μ is Population mean. S is the standard deviation of Population and n is sample size.

If the standard deviation of the sample 's' is given then t-Statistic is defined as $\boxed{t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}}}$

Note: The relation between s and S is $ns^2 = (n-1)S^2$.

Imp.

Application of t - Distribution :-

Some of the applications of t - distribution are given below:

1. To test if the sample mean ($\bar{x}$) differs significantly from the hypothetical value μ of the Population mean.
2. To test the significance between two sample means.
3. To test the significance of observed Partial and multiple Correlation coefficients.

Question !→ A random sample of size 16 has 53 as mean. The sum of squares of the deviation from mean is 135. Can this sample be regarded as taken from the Population having 56 as mean? obtain 95% and 99% confidence limits of the mean of the Population.

Solution :- Null Hypothesis, H_0: There is no significant difference between the sample mean and hypothetical Population mean i.e., $\mu = 56$.