

Class Batch 2nd yr (ECE)
Date: 8/5/2020
Topic: Signal & System
Instructor: Mohan Singh

Topic: Property of Fourier Series / Transform

1. Linearity

$$\text{If } x_1(t) \xrightarrow{F} X_1(f)$$

$$\text{and } x_2(t) \xrightarrow{F} X_2(f)$$

then

$$a_1 x_1(t) + a_2 x_2(t) \xrightarrow{F} a_1 X_1(f) + a_2 X_2(f)$$

This follows directly from the definition of the Fourier transform (as the integral operator is linear). It is easily extended to a linear combination of an arbitrary number of signals.

② Time scaling:-

* Let $x(t)$ and $X(f)$ be Fourier Transform pairs and let " α " be a constant. Then time scaling property states that:

$$x(\alpha t) \xleftrightarrow{F} \frac{1}{|\alpha|} X\left(\frac{f}{\alpha}\right)$$

$X(\alpha t)$ represents a time scaled signal and $X\left(\frac{f}{\alpha}\right)$

represents frequency scaled signals.

* For $\alpha < 1$, $X(\alpha t)$ represents compressed signal but $X\left(\frac{f}{\alpha}\right)$ represents expanded

version of $X(f)$.

* For $\alpha > 1$, $X(\alpha t)$ will be expanded signal in the time domain. But its fourier transform $X\left(\frac{f}{\alpha}\right)$

represents versions of $X(f)$.

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Time Shifting

* The time shifting property states that if $x(t)$ and $X(f)$ form a fourier transform pair then

$$x(t - t_d) \xleftrightarrow{F} e^{-j2\pi f t_d} \cdot X(f)$$

* It is the same signal $x(t)$ only shifted in time

here the signal $x(t - t_d)$ is time shifted signal.

4. Duality or Symmetry:

* This property states that

$$\text{if } x(t) \xrightarrow{F} X(-f)$$

then

$$X(t) \xrightarrow{F} x(-f)$$

* The duality theorem tells us that the shape of the signal in the time domain and the shape of the spectrum can be interchanged.

Area under $x(t)$

* This property states that the area under the curve $x(t)$ equals the values of its Fourier transform at $F=0$.

i.e. if $x(t) \xleftrightarrow{F} X(f)$.

then $x(t) = X(0)$

Area under $X(f)$

* This property states that the area under the curve $X(f)$ equals the values of signal $x(t)$ at $t=0$.

if $x(t) \xrightarrow{F} X(f)$

then $x(t) = X(f)$

Frequency shifting

- * The frequency shifting characteristic states that if $x(t)$ and $X(f)$ form a fourier transform pair then

$$e^{j2\pi f_c t} x(t) \xrightarrow{F} X(f - f_c)$$

- * f_c is a real constant

Differentiation

In Time Domain

* This property is applicable if and only if the derivative of $x(t)$ is fourier transformable.

$$\frac{d}{dt} x(t) \xrightarrow{F} j2\pi f X(f)$$

Integration in Time Domain

* Integration in time domain is equivalent to dividing the fourier transform by $(j2\pi f)$

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* If $x(t) \xleftrightarrow{F} X(f)$ and provided that $X(0) = 0$

$$\int_{-\infty}^t x(\lambda) d\lambda \xleftrightarrow{F} \int_{2\pi f} X(f)$$

Multiplication in Time Domain

* The multiplication theorem states that:

If $x_1(t) \xleftrightarrow{F} X_1(f)$ and

$x_2(t) \xleftrightarrow{F} X_2(f)$ are

the two fourier transform pair then

$$x_1(t) \cdot x_2(t) \xrightarrow{F} \int_{-\infty}^{\infty} X_1(\tau) \cdot X_2(t-\tau) d\tau$$

This means that multiplication of two signals in time domain gets transformed into convolution of the fourier transform

$$x_1(t) \cdot x_2(t) \xrightarrow{F}$$

$$X_1(f) * X_2(f)$$

Planning for

to

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Class Work Forecast

Home Work

Convolution in Time Domain

This property states that the convolutions of signals in the time domain will be transformed into the multiplications of their Fourier transform in the frequency domain

i.e.

$$x_1(t) * x_2(t) \xrightarrow{F}$$

$$X_1(f) \cdot X_2(f)$$