Class - B.Tech (ECE) 2nd yr.
Sub: Signal & Systems
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Topic: Numerical

Topic: Use of DFT in linear filtering.

Linear filtering is same as linear convolutions. In this section we will discuss how the linear convolution is obtained using DFT. We know that the linear convolutions of x(n) and h(n) is given by:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k). \quad (1)$$

If we obtain fourier transform of x(n) and h(n) then we will get $X(\omega)$ and $H(\omega)$. We know that convolutions is equivalent to multiplication in the frequency domain. So by multiplying $X(\omega)$ and $H(\omega)$ we will get $Y(\omega)$

$$Y(\omega) = X(\omega) \cdot H(\omega). \quad (2)$$

and

This sequence will be same as linear convolutions of x(n) and h(n). But we cannot use fourier Transform to obtain linear Convolutions because of following reasons:

1. In fourier transform "ω" is Continuous functions of frequency. So the Computation

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computations cannot be done
on digital computers Because
for the digital signal processor
we want discrete signals
and not the continuous signal

2) If we use DFT then the
computation will be more
efficient because of
the availability of fast
fourier tranform (FFT) algorithm.

So we must use DFT to
obtain linear filtering operation
In case of DFT we have
studied that the multiplications
of two DFT in frequency
domain is equivalent
to the circular convolutions
that means

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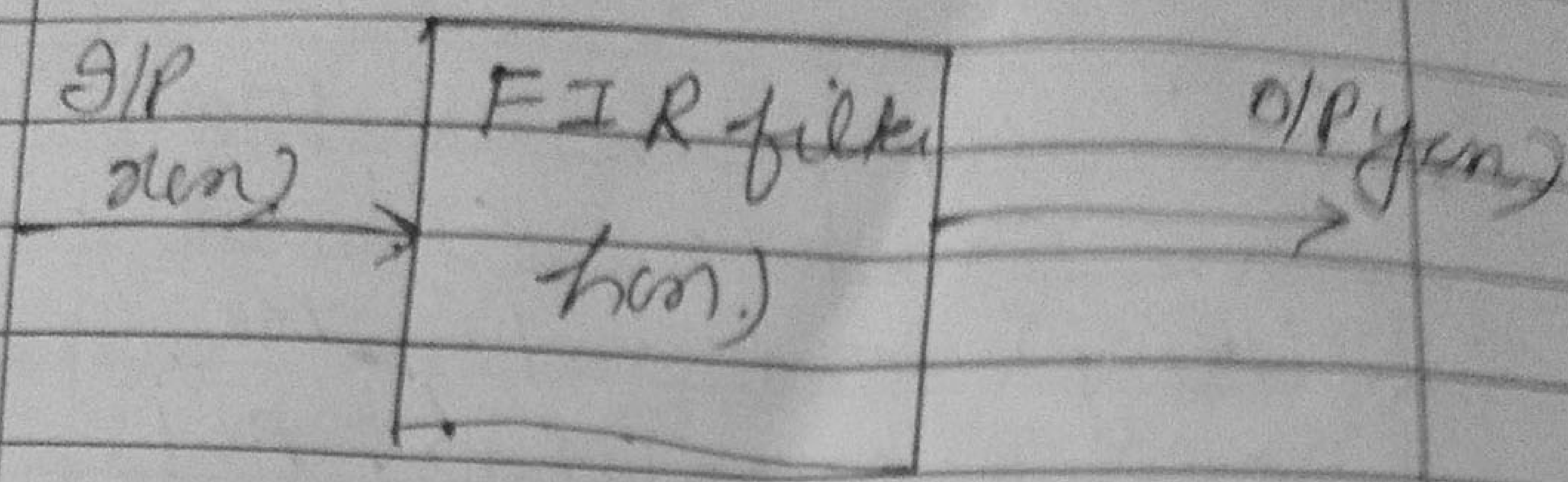
$$X(k) \cdot H(k) = x(n) \, (N) \, h(n)$$

But in this case, we want linear convolutions (linear filtering) and not the circular convolutions. But if we adjust the length of two sequence x(n) and h(n), then the same result can be obtained using linear convolutions and circular convolutions.

Consider an FIR filter having impulse response h(n) as shown in fig



here $x(n)$ = I/P sequence having length "L".

$\therefore x(n) = \{0, 1, 2 \text{ ---- } L-1\}$.

$h(n)$ = Impulse response of filter having length "M".

$\therefore h(n) = \{0, 1, 2 \text{ --- }, M-1\}$

Hence the linear convolutions of $x(n)$ and $h(n)$ produces the output sequences $y(n)$ and the length of $y(n)$ is.

$$\boxed{N = L + M - 1.} \quad \text{--- (3)}$$

In this case both the sequence $x(n)$ and $h(n)$ are finite so linear convolutions will be finite thus eq. (1) becomes

$$y(n) = \sum_{k=0}^{M-1} h(k) \cdot x(n-k);$$

Now if we adjust the length of $x(n)$ and $h(n)$ equal to "N" and if we perform the circular the circular convolution of x(n) and h(n) then the result will be same as linear convolution. The lengths of x(n) and h(n) can be made equal to "N" by adding required numbers of zero in x(n) and h(n). This is called as "zero padding". That means we have to increase the lengths of x(n) by M points and lengths of h(n) by L points to make the total lengths $N = L + M - 1$. Then we can obtain the DFT of x(n) and h(n) that is $X(k)$ and $H(k)$.

The multiplications of these two DFT gives sequence. $Y(k)$

$$\therefore Y(k) = X(k) \cdot H(k)$$

Now by taking I.DFT of $Y(k)$. the o/p sequence $y(n)$ can be obtained. Thus the linear filtering can be obtained.

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