DATE

Question → $x[n] = \cos an \; u[n]$

(i) Draw the signal.

(ii) Determine D.T.F.T of given Signal

Sol^n → (ii) We know that $\cos an = \dfrac{e^{jan} + e^{-jan}}{2}$

Then D.T.F.T $\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$$= \sum_{n=-\infty}^{-1} x[n] e^{-j\omega n} + \sum_{n=0}^{\infty} x[n] e^{-j\omega n}$$

$$= 0 + \sum_{n=0}^{\infty} \left(\frac{e^{jan} + e^{-jan}}{2} \right) e^{-j\omega n}$$

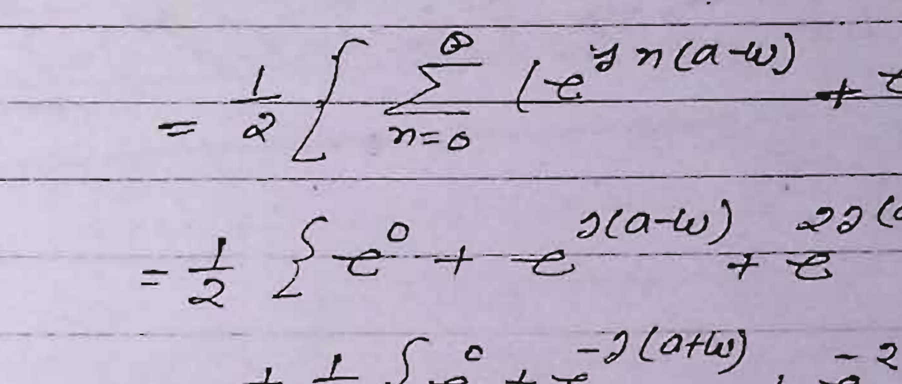
$$= \frac{1}{2} \left\{ \sum_{n=0}^{\infty} \left(e^{jn(a-\omega)} + e^{-jn(a+\omega)} \right\} \right.$$

$$= \frac{1}{2} \left\{ e^{0} + e^{j(a-\omega)} + e^{2j(a-\omega)} + \cdots \right\}$$

$$+ \frac{1}{2} \left\{ e^{0} + e^{-j(a+\omega)} + e^{-2j(a+\omega)} + \cdots \right\}$$

$$\text{So} \quad = \frac{1}{2} \left\{ \frac{1}{1 - e^{j(a-\omega)}} + \frac{1}{1 - e^{-j(a+\omega)}} \right\}$$

(i)



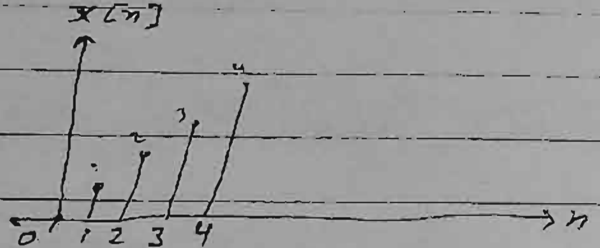
Question → $x[n] = n\, u[n]$

(i) Sketch the Signal ?

(ii) Determine D.T.F.T of given Signal by basic definition ?

Sol^n → (i)

Unit Ramp Signal



(ii) We Know that $D.T.F.T\{x[n]\} = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

Then $D.T.F.T\{n\, u[n]\} = \sum_{n=-\infty}^{\infty} n\, u[n] e^{-j\omega n}$

$$= \sum_{n=-\infty}^{-1} n\, u[n] e^{-j\omega n} + \sum_{n=0}^{\infty} n\, u[n] e^{-j\omega n}$$

$$= 0 + \sum_{n=0}^{\infty} n\, u[n] e^{-j\omega n}$$

$$= 0 + \{0 + 1\, e^{-j\omega} + 2(e^{-j\omega})^2 + 3(e^{-j\omega})^3 + 4(e^{-j\omega})^4 + \cdots$$

Let $S = 1 \cdot e^{-j\omega} + 2(e^{-j\omega})^2 + 3(e^{-j\omega})^3 + 4(e^{-j\omega})^4 + \cdots$

$S(e^{-j\omega}) = \quad (e^{-j\omega})^2 + 2(e^{-j\omega})^3 + 3(e^{-j\omega})^4 + \cdots$

$S - S(e^{-j\omega}) = e^{-j\omega} + (e^{-j\omega})^2 + (e^{-j\omega})^3 + (e^{-j\omega})^4 + \cdots$

$$S(1-e^{-jw}) = e^{-jw} + (e^{-jw})^2 + (e^{-jw})^3 + \dots \infty$$

$$S(1-e^{-jw}) = \frac{e^{-jw}}{1-e^{-jw}} \qquad \left\{ S_\infty = \frac{a}{1-r} \right.$$

$$S = \frac{e^{-jw}}{(1-e^{-jw})^2}$$

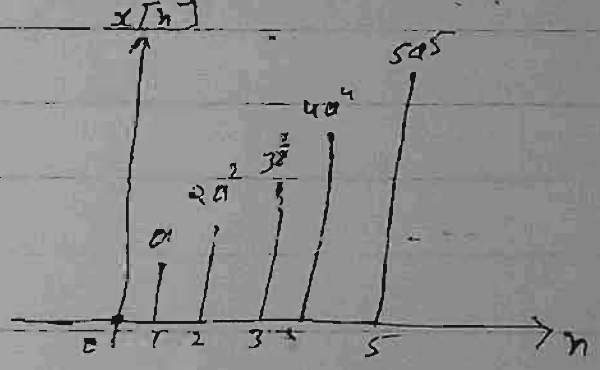
Question → $x[n] = n a^n u[n]$

(i) Sketch the Signal.

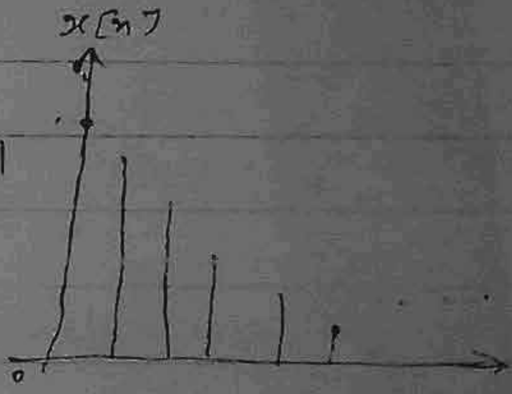
(ii) Determine the D.T.F.T of given Signal by basic definition.

Sol^n →

$0 < a > 1$



$a < 1$



(ii) We know that, $D.T.F.T\{x[n]\} = X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}$

$$D.T.F.T\{n a^n u[n]\} = \sum_{n=-\infty}^{\infty} n a^n u[n]\, e^{-jwn}$$

$$= \sum_{n=-\infty}^{-1} n a^n u[n] e^{-jwn} + \sum_{n=0}^{\infty} n a^n u[n] e^{-jwn}$$

$$= 0 + \{0 + a e^{-jw} + 2a^2 (e^{-jw})^2 + 3a^3 (e^{-jw})^3 + \dots\}$$

Let $S = a e^{-jw} + 2a^2 (e^{-jw})^2 + 3(a e^{-jw})^3 + \dots \infty$

$S(a e^{-jw}) = (a e^{-jw})^2 + 2(a e^{-jw})^3 + \dots \infty$

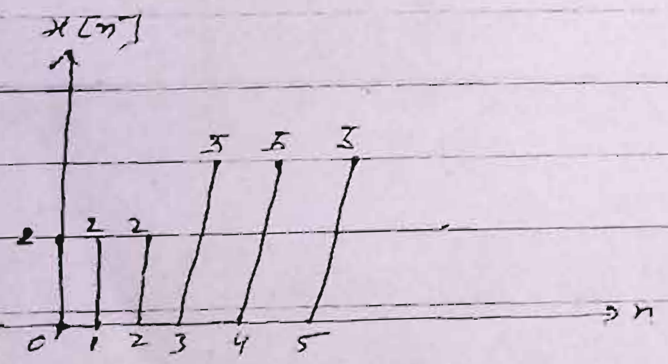
$S - S(a e^{-jw}) = a e^{-jw} + (a e^{-jw})^2 + (a e^{-jw})^3 \dots \infty$

$$S(1 - a e^{-jw}) = \frac{a e^{-jw}}{1 - a e^{-jw}} \qquad \{ S_\infty = \frac{a}{1-r}$$

$$S = \frac{a e^{-jw}}{(1 - a e^{-jw})^2}$$

Ans

Question →



Determine D.T.F.T of This Signal.

PAGE NO
DATE

$Sol^n \Rightarrow$ We know the Definition of This Signal -

$$u[n] = \begin{cases} 2 & 0 < n \geq 2 \\ 5 & 3 < n \geq 5 \end{cases}$$

Then $D \cdot T \cdot F \cdot T \{x[n]\} = X(e^{jw}) = \sum_{n=\infty}^{\infty} x[n] e^{-jwn}$

$$D \cdot T \cdot F \cdot T \{x[n]\} = \sum_{n=0}^{5} x[n] e^{-jwn}$$

$$= \sum_{n=0}^{2} x[n] e^{-jwn} + \sum_{n=3}^{5} x[n] e^{-jwn}$$

$$= \sum_{n=0}^{2} 2 e^{-jwn} + \sum_{n=3}^{5} 5 \cdot e^{-jwn}$$

$$\Rightarrow \left\{2 + 2e^{-jw} + 2(e^{-jw})^2\right\} + \left\{5(e^{-jw})^3 + 5(e^{-jw})^4 + 5(e^{-jw})\right\}$$

$$= \left\{\frac{2\left[1 - (e^{-jw})^3\right]}{1 - e^{-jw}}\right\} + \left\{\frac{5(e^{-jw})^3\left[1 - (e^{-jw})^3\right]}{1 - e^{-jw}}\right\}$$

$$= \frac{\left[1 - (e^{-jw})^3\right]}{1 - e^{-jw}} \left\{2 + 5(e^{-jw})^3\right\}$$

Question → $x[n] = a^{|n|} = \begin{cases} a^{-n} & n<0 \\ a^{n} & n\geq 0 \end{cases}$

(i) Sketch the Signal.

(ii) Find D.T.F.T of given Signal.

Sol^n → We Know that DTFT of $x[n] = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$$= \sum_{n=-\infty}^{-1} x[n] e^{-j\omega n} + \sum_{n=0}^{\infty} x[n] e^{-j\omega n}$$

or

$$= \sum_{n=-1}^{-\infty} x[n] e^{-j\omega n} + \sum_{n=0}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-1}^{-\infty} a^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} a^{n} e^{-j\omega n}$$

$$= \left\{a e^{j\omega} + a^2 e^{j2\omega} + \cdots \infty\right\} + \left\{1 + a e^{-j\omega} + a^2 e^{-j2\omega} + \cdots\right\}$$

$$= \frac{a e^{j\omega}}{1-a e^{j\omega}} + \frac{1}{1-a e^{-j\omega}} \qquad \left\{S_\infty = \frac{a}{1-r}\right\}$$

(i)